

# **Manual for Equations<sup>®</sup> and On-Sets<sup>®</sup> Judges**

**2018-19 Edition**

**by**

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### **On-Sets Situations**

This document is a separate download.

#### Guidelines for Judging the Correctness of On-Sets Solutions

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## Preliminary Remarks

1. This manual presents situations that may arise in an Equations or On-Sets match and the ruling that an official should make in each case. Each item takes this form:
  - Any *variations* in force are listed;
  - the *situation* is stated;
  - the *ruling* is given;
  - sometimes a *penalty* is listed;
  - occasionally a *comment* is added.
2. This manual is intended to supplement to the *Tournament Rules for Equations and On-Sets*. Many situations covered explicitly in the *Tournament Rules* are not reiterated here.
3. While directly aimed at judges, this manual can also help train players and coaches.
  - Players who read through the situations cannot help but learn the rules better and become better players.
  - When players know the rules thoroughly, judges are not needed.
4. Readers may disagree with rulings stated herein. Also, situations may develop that have not been explicitly covered.
  - The writer asks all who use these pages to communicate any disagreements, questions, or new situations to him, so they can be considered for future editions.
5. Some situations apply only to certain divisions.
  - For example, a situation marked “E only” applies just to Elementary Division and one marked “MJS only” affects just Middle, Junior, and Senior Divisions, and so on.
6. A continuing topic of discussion has been the role of a judge when an opponent objects to a Solution but does not give a specific reason for rejecting it.
  - General guidelines for judges asked to rule on the mathematical correctness of a Solution have been developed for On-Sets and are included with the On-Sets situations in a separate download.
  - The sections of the *Tournament Rules* for checking Solutions in both Equations and On-Sets have been expanded to make more explicit the procedures to be followed by players in determining whether a Solution is correct.
7. This manual is up-to-date as of the 2018-19 school year.
  - Any rules changes after the date of publication are not reflected in these pages. However, the Manual will be updated yearly to handle rules changes and any additional situations that arise during tournament play.
  - So you should download the latest version of this manual each year just as you do for the *Tournament Rules*.
  - As with the *Tournament Rules*, changes or additions from the previous year are printed **bold**.
8. The author thanks the many teachers and students who, since 1966, have helped in the compilation of this manual. In particular, Craig Zeller has reviewed the Manual and suggested numerous clarifications. However, the author alone takes responsibility for any mistakes or inconsistencies in this manual.

## Some General Comments on Judging

The following guidelines represent a consensus of experienced judging from many leagues. These guidelines can form the basis for training sessions for judges. Any questions about or suggestions for improving these guidelines are welcome.

1. An official should never rule *before* players have taken a stand on an issue.
  - *Exception:* Players may ask a question for their own reference. A judge may answer it if *all* at the table want to hear the explanation.
  - A judge's ruling should never give unfair advantage to one or two players by helping them to decide whether to challenge, charge illegal procedure, change a Solution, whom to join on a challenge, etc.
  - If a player requests a ruling because he does not know what to do, the official should tell the player to take a stand first before giving the correct interpretation of a situation. Sometimes, you may have to wait until Solutions are presented.
  - If an official inadvertently answers a question that helps a player decide what to do, the official should consult with other judges and, if necessary, order the shake re-played.
2. When an official is called to a table, *the timer should be laid down* while the official learns the situation and makes a ruling.
  - Further, **during the last five minutes** of a round before the warning, the official should note how long settling a controversy takes and **add that amount to the warning time for that table.**
  - This practice prevents the player in the lead from stalling by creating or prolonging a controversy just before the warning.
3. Many conflicts arise because players do not express themselves clearly.
  - *Examples:* A player may be charging illegal procedure without using the term "illegal procedure," and the opponent is not sure what to do. Or a player is in effect challenging without clearly stating what challenge is being made.
  - *Often the primary objective of a judge is to facilitate communication among the players.* A judge may have to ask a player point blank: "Are you challenging the move or charging illegal procedure?" Or the judge may have to translate a player's objection into language the opponent can understand.
  - All this must be done without putting ideas into players' heads. For example, a judge must not tell a player he should challenge but, if the player is in effect saying that a Solution can be written with only one more cube, the judge could rule that the player is challenging Now.
4. Players may give conflicting testimony.
  - *Examples:* Was a ten-second warning given? Did the cube touch the mat? Was the cube in Required or Permitted? etc.
  - In a two-player match, there is no way for a judge to settle a conflict in testimony. The only rule of thumb is not to penalize a player unless the evidence is *definite*. For example, one player in a two-player match says a ten-second countdown was given, but the Mover says it wasn't or was counted down at too fast a pace. Do not penalize the Mover.
  - In a three-player match, if two of the three agree on a situation, the judge can follow the majority opinion. But if the third player does not know or refuses to take a stand on what happened, the judge is back to the two-way situation described above.

- In rare cases, as a last resort, a shake may have to be replayed.
5. Once all players in a match have agreed that the last Solution to be checked is correct (or incorrect), the shake is finished and no appeal to a judge is possible.
    - Sometimes a player later realizes he overlooked a mistake in an opponent's Solution.
    - If he (and the third player if there was one) accepted the Solution, it is too late to reverse the outcome of that shake.
  6. A different issue from #5 is a question about the *scoring* of a shake.
    - A player may appeal the scoring of any shake in a round up to the time when he initials the scoresheet.
    - Thus, a player who realizes that an earlier shake of the match was scored wrong may appeal to a judge and have the scoring corrected before initialing the scoresheet.
  7. A player who disagrees with a judge's ruling may request a **second opinion**.
    - The player should tell the first judge that she would like a second opinion. In this way, the first judge can tell the next judge that a second opinion is being given without giving the second judge any particulars of the situation that would prejudice the second ruling.
    - After making a ruling, the second judge should ask the players if the second ruling agreed with the first. If it did, *the player may seek no other judges* but still may protest the decision. If the two judges disagree, a third judge should be enlisted without giving her any particulars of the situation.
    - The intent is to prevent a player from "shopping" for a favorable ruling. The player unhappy with a ruling can protest at the end of the round. In the meantime, play resumes at the table. *Do not allow a player to demand a third judge. If she refuses to move on to the next shake, start it without her.*
    - In tricky situations, a *group* of judges can hear the situation and decide on a ruling. If judges are discussing a situation that occurred at a table, the players should continue play or move to the next shake if possible. For example, if judges are deciding whether a Solution is correct and no other Solutions are in dispute, the players can move to the next Solution or the next shake.
  8. A player may **appeal** a ruling only **if a second judge was called, and the player does not initial the scoresheet to accept the final scores**.
    - The protesting player must complete and submit a Protest Form as soon as possible at the end of the match and submit any relevant documents (such as the paper on which a disputed Equation/Solution was written). The names of the two (or three) judges who ruled on the situation must be written on the form.
    - The Appeals Panel that adjudicates the protest then has the needed information, although the parties involved (including any judges called to the table) may be contacted to answer questions.
    - The rule followed by this "higher authority" (individual or board) will be to *uphold the decision* made on the spot unless the ruling *clearly violates* a rule or interpretation in the Tournament Rules or this manual.
  9. In general, judges may tell the players the following.
    - a. how to score a shake;
    - b. how much time remains in the match (round);

- c. whether a disputed move is illegal procedure;
- d. if a variation or combination of variations allows a player to take a certain action or use a cube in a specific way;
- e. if a variation chosen by a player is legal or if two variations chosen for a shake are contradictory and, if so, how the situation is to be resolved;
- f. how much time a player has to complete a certain action;
- g. exactly what a player is charging or challenging (after hearing the explanation and any relevant testimony from the other parties);
- h. how to score the match (round).

### Timing

The rules allot two minutes for checking a Solution. Yet the checker(s) may obtain additional time, without suffering a one-point penalty, by calling a judge. To avoid abuses of the timing rule, the following guidelines are offered.

1. If a checker has a specific question about the Solution and the two minutes for checking have not expired, the timer should be turned down while the judge is called and answers the specific question.
  - If the Solution is not rejected as a result of the judge's answer, timing should be immediately resumed.
  - Players may call the judge again to settle any dispute about the mathematical correctness of the Solution.
2. Suppose the checker calls the judge to answer a specific question just as the time limit for checking a Solution expires.
  - If the judge's answer does not invalidate the Solution, the checker may continue checking the Solution for an additional minute *only by taking a one-point penalty*.
  - See #5 below for the situation where the third player does not want to continue checking.
3. The checker has worked out the Solution and obtains a different answer from the writer.
  - The judge oversees the working out of the Solution in order to decide whether it equals the Goal.
  - If it does, the checker(s) must accept the Solution since they did not voice any other objection to it.
  - For example, if a checker notices that the Solution does not contain a cube in Required while the Solution is being checked in the presence of the judge, it is too late to point this out since the time limit for checking has expired.
4. Suppose the time limit for checking expires, and a checker calls the judge without having first checked the Solution using the cubes.
  - *The judge should not work out the Solution nor oversee the working out of the Solution.*
  - Instead the checker(s) must either accept the Solution or pay a one-point penalty to obtain an additional minute to work out the Solution.
  - If a checker works out the Solution during the extra minute and obtains a result not equal to the Goal, the judge may be recalled to settle the dispute in accordance with the guidelines stated above.
5. Two players are checking a Solution. When the two-minute time limit expires, one player accepts the Solution. The other takes a -1 to obtain an additional minute.

- *During this extra minute, the player who already accepted the Solution may not participate in the checking.*
- If the player who accepted the Solution points out an error during the additional time taken by the other checker, that player is also penalized one point for going into the extra minute for checking.

### **Scoring**

The following rule is included in the Tournament Rules to make the scoring more fair in the case where one opponent accepts a Solution but the other opponent finds an error in it.

*A player who did not present a Solution scores 2 for the shake if that player accepts another player's Solution as correct even if that Solution is subsequently proved wrong by another checker.*

## General Principles Applied to Situations in This Manual

1. Each variation selected must have the potential to affect the shake and must not conflict with a previous player's variation.  
*Example* Choosing 0 wild with no 0's in Resources is a "wasted" selection.
2. A player has not selected a variation until the player circles the name of the variation on the selection sheet.
3. It is helpful if the Goal-setter says "Goal" or "Goal Set" when finished. However, there are other ways the Goal-setter can indicate that the Goal is complete. (The same principle applies to dealing the Universe in On-Sets.)  
*Examples* Starting the stall for the next player, allowing the next player to move, in On-Sets placing extra digit cube(s) in Forbidden
4. An illegal procedure is insulated by a legal action by a subsequent player. You may not insulate your own illegal procedure.  
*Example* The player leading in the match makes a Bonus move, but the next player makes a legal move or challenges.
5. No time penalty can be enforced if the player was not given a ten-second warning that began when the sand ran out and was conducted at a reasonable pace.
6. A Mover (including the Goal-setter) may not specify what a wild cube or other cube with multiple meanings represents for all players. However, the placement of a cube in the Goal may limit its interpretation.  
*Example* With 0 wild, the Goal-setter may not specify what a 0 in the Goal must equal. (**MJS** only) The placement of the 0 in the Goal may determine whether it must be a numeral or an operation.
7. A player attempting to challenge must pick up (not just touch) the challenge block. On the other hand, players must not pick up the challenge block for any other purpose (such as saying "Goal" or charging illegal procedure).
8. A legal challenge "freezes" the mat. No more moves may be made for this shake.
9. Any challenge is automatically directed against the latest Mover.
10. Some actions are both illegal procedure and make a Solution possible or impossible. In this case, the first charge that is made takes priority. If an opponent charges illegal procedure before the other opponent challenges, handle the situation as illegal procedure. If an opponent challenges before another charges illegal procedure, work out the challenge in the usual way, ignoring the illegal procedure call.  
*Example* The player leading in the match makes a Bonus move to Forbidden that makes all Solutions impossible.
11. A wild 0 cube is ambiguous for upside-down depending on its placement in the Goal. (**JS** only) A wild x cube is ambiguous for both upside-down and sideways in the Goal, and \* or ^ (for ten in base eleven or twelve) is ambiguous for upside-down and sideways in the Goal. (^ may not be placed sideways or upside-down in the Goal since ^ behaves the same as \*.)
12. A Solution-writer must make sure that his Solution is unambiguous and, if the Goal is ambiguous, equals the interpretation of the Goal that the writer writes with the Solution.
13. (**MJS** only) When Base  $m$  is in effect, the Goal should be converted to base ten before any other variations are applied in computing the Goal's value(s).



## Section A: Beginning a Shake and Selecting Variations

*NOTE* Situations **A1-A21** (with appropriate modifications) also apply to On-Sets.

- A1** Situation The players must decide who will be the first Goal-setter of the match (round).  
Ruling Each player rolls a red cube. Highest digit goes first. A player who rolls an operation sign is eliminated unless all players roll an operation sign. Players tied for high digit roll again until the tie is broken.  
Comment In On-Sets, each player rolls a digit cube.
- A2** Situation The Goal-setter rolls the cubes. However, when the minute for rolling the cubes expires, the Goal-setter has not circled a variation. An opponent wants to penalize the Goal-setter one point.  
Ruling The Goal-setter has 15 seconds after the minute for rolling the cubes to select a variation.  
Penalty None  
Comment If the Goal-setter has not selected a variation at the end of the minute for rolling the cubes, an opponent should start a 15 second countdown.
- A3** Situation The Goal-setter rolls the cubes and selects a variation before the one minute for rolling the cubes has expired. The next player is then given 15 seconds to select a variation.  
Ruling This is not correct. The next player still has the remainder of the minute for rolling the cubes plus 15 more seconds to select a variation. If the next player also selects a variation before the minute ends, the third player (if there is one) has the remainder of the minute plus 15 seconds to select a variation.  
Comment If the Goal-setter was not timed for rolling the cubes, the second player may demand a full minute plus fifteen seconds before selecting a variation.
- A4** Situation A player announces a variation but has not circled it on the sheet.  
Ruling The player must circle the name of the variation within 15 seconds after the previous player selected (or, for the Goal-setter, after the one minute for rolling the cubes expires).  
Penalty The player failing to circle a selection within 15 seconds loses a point. If, after another 15 seconds, the player still has not circled a selection, the player loses another point and may not pick a variation for that shake.
- A5** Situation A player states one variation but circles another on the sheet. The discrepancy is not noticed until Solutions are presented.  
Ruling If there is evidence the player intentionally misled opponents and the variation choice has affected the shake, order the shake replayed and, **with the consent of a second judge**, penalize the player a point. Otherwise, the variation the player *circled* is in force, not the one stated orally.  
Comment MJS: This ruling also applies to a situation where, for example, a player calls Base 8 but writes a 9 in the blank after "Base" on the selection sheet.
- A6** Situation A player states the name of a variation that is illegal for the shake; for example, 0 wild when no 0 was rolled. However, the player realizes the mistake and circles a legal variation on the scoresheet.  
Ruling What counts is what is circled. Ignore the statement of the illegal variation.  
Penalty None unless the 15-second time limit expired before circling the selection.

- A7** Situation (MJS only in Equations and all divisions in On-Sets) A player circles a variation like Base or Multiple of  $k$  but does not fill in which base or which  $k$ , or, in On-Sets, which wild cube without specifying which cube is wild, etc.
- Ruling The player must complete the variation selection by filling in the base, value of  $k$ , etc., within 15 seconds (even if the player stated the value orally).
- Penalty -1 if the player does not complete the selection within 15 seconds
- A8** Situation After the cubes are rolled, a player turns one over or in some way interferes with players' view of the Resources.
- Ruling Once the cubes are rolled, no player may alter the face of the cubes nor obstruct other players' view of any cubes remaining in Resources. If two out of three players agree that a cube was turned over, reset it to its original position. Warn the player who interfered with the Resources that any repetition of this action will result in a one-point penalty.
- Penalty If this is the second time that player has interfered with the cubes during the round or the player has had another warning under the Behavior Rule (IX- C), penalize the player one point. However, if there is evidence that the player's turning over the cube was intentional to gain an advantage – as when turning over a cube to provide a symbol needed in the Solution that wasn't available – two or more judges may decide on a stronger penalty for the player even to the point of disqualification.
- Comment In Elementary On-Sets, the Goal-setter must set out two  $\nabla$  cubes and one  $\triangle$  cube (or vice-versa) before rolling. If the Goal-setter forgets and rolls the cubes, then any = or  $\subseteq$  must be turned over to  $\nabla$  and  $\triangle$  when the players realize the mistake.
- A9** Situation One player circles the variation selections for all players at the table.
- Ruling This is not the intended procedure for selecting variations. The Goal-setter circles his selection, then passes the sheet to the next player for selection, and so on. If there is no argument over what variations were circled, simply tell the players to follow the correct procedure for subsequent shakes.
- Penalty None unless there is evidence that the player circling for other players did not circle the variation an opponent intended to select. In that case, warn the player that any repetition of this action will result in a one point penalty.
- A10** Situation A player selects an illegal variation, but no opponent notices this. For example, a player chooses 0 wild with no 0 in Resources. The next player chooses a legal variation, the Goal-setter makes a legal bonus move, or the Goal is set.
- Ruling The illegal procedure of the first player is insulated. However, the faulty variation selection has no effect on the shake. Ignore it.
- A11** Situation Before the cubes are rolled, a player selects a variation.
- Ruling Illegal procedure; set aside the variation selection. The Goal-setter must roll the cubes before variations are selected. Since the order of play has been disrupted, reset the time for rolling the cubes to one minute.
- Comment The ruling also applies to an On-Sets player selecting a variation before the cards are dealt.

- A12** Situation Before one or more players select a variation, the Goal-setter sets a Goal.  
Ruling Illegal procedure; this is similar to moving out of turn. The cubes on the Goal section (and any bonus cube in Forbidden) are returned to Resources. Any player who has not chosen a variation is allowed to make a selection. Then the Goal-setter has two minutes to set a Goal.
- A13** Situation Same as **A12** except that, before anyone charges illegal procedure against the Goal-setter, an opponent issues an Impossible challenge against the Goal-setter.  
Ruling The challenge insulates the illegal procedure. The Goal stands as played, and the challenge is worked out in the usual way. Any player who has not selected a variation may not choose one for this shake.
- A14** Situation Same as **A12** except that, before anyone charges illegal procedure against the Goal-setter, the next player moves a cube to Required, Permitted, or Forbidden.  
Ruling The move insulates the illegal procedure. The players who were not given a chance to select a variation may not choose one for this shake.
- A15** Situation The Goal-setter, while ahead in the match, makes a bonus move, then sets the Goal. An opponent charges illegal procedure against the bonus move.  
Ruling You may not insulate your own illegal procedure. So return the cube in Forbidden to Resources. The Goal stands as set, unless the Goal-setter did not say "Goal" or in some way indicate the Goal was finished. In the latter case, the Goal-setter may revise or add to the Goal, although all cubes played to the Goal section of the mat must stay there.  
Penalty The Goal-setter loses one point for making a bonus move when ahead. The Goal-setter may also lose a point if not finished setting the Goal before the two-minute time limit expires.
- A16** Situation After variations have been selected and/or the Goal has been set, another player rerolls the cubes, including any cubes in the Goal.  
Ruling Restart the shake with the same player as Goal-setter. This player rolls the cubes again. Add the time taken for the aborted shake to the time limit for the round for that table.  
Penalty If there is evidence that the player intentionally rerolled the cubes (for example, he was told not to but did so anyway), penalize the player one point, and issue a warning that any additional disruption will result in stronger penalties.
- A17** Situation In a two-player match or in any Senior match, a player chooses two variations. However, one of them is an illegal choice (e.g., 0 wild when no 0 is in Resources). The player argues that, since one of the two variations selected was legal, a one-point penalty should not be enforced.  
Ruling The argument is incorrect. A player selecting two variations must choose two legal variations. The player must pick another variation within 15 seconds to replace the illegal one.  
Penalty -1 for the player who made the illegal selection
- A18** Situation A player chooses two variations. However, both of them are illegal (e.g., 0 wild when no 0 is in Resources and average when no + was rolled). Is the player penalized two points and prohibited from picking a variation for the shake?

- Ruling No, the player has made one illegal *try* at making his variation selection. So he loses one point and makes another selection of either one or two variations. If the second selection is illegal, he loses a second point and may not pick a variation for the shake.
- A19** Situation A player has asked for a second opinion on an Equation or other matter and received the same ruling from both judges. However, the player insists on a third opinion and refuses to move on to the next shake.
- Ruling Order the next shake to begin. If the player who asked for a third judge refuses to continue, disqualify him or her from the match. If appropriate, add time to the match to make up for lost minutes spent haggling over obtaining a third opinion.
- A20** Situation (**EMJ** only) In a two-player match, the player who is not the Goal-setter does not choose two variations within 15 seconds.
- Ruling Illegal procedure
- Penalty The player loses one point and has 15 seconds to select another variation.
- Comment The player who must choose two variations must select (circle) both within the 15-second time limit.
- A21** Situation (**EMJ** only) In a two-player match, the Goal-setter circles two variations.
- Ruling Illegal procedure; the second variation selection is invalid and is deleted.
- Penalty None
- A22** Situation (**M** only) One player selects 0 wild. Then someone chooses average even though no + is in Resources. Is the second selection legal?
- Ruling Yes, it is. Since 0 may be used for +, the second variation may affect the shake. If the same player chooses both variations, the order in which they are circled makes no difference.
- A23** Situation (**MJS** only) A player selects a variation like “Base six.” However, no opponent notices the error, and the next player selects a legal variation or the Goal is set.
- Ruling The faulty variation selection is insulated but has no effect on the shake. That is, Solutions are in base ten, *not* base six.
- A24** Situation (**MJS** only) Same as **A22** except that, after the Goal is set, a player challenges Impossible.
- Ruling The Goal is not automatically impossible because of the illegal variation. Since the illegal variation is ignored, Solutions are judged according to the legal variations that were chosen. The challenge stands and is worked out in the usual manner.
- A25** Situation (**MJS** only) Same as **A22** except that the Goal-setter declares “no Goal.”
- Ruling Since the illegal variation is ignored, the no Goal declaration is probably erroneous. An opponent may disagree with the declaration and set a Goal. If anyone challenges Impossible against this Goal, the Goal-setter can write a Solution that ignores the illegal variation.
- Penalty If the no Goal declaration is successfully challenged, the original Goal-setter loses two points.

- A26** Situation (MJS only) One player selects 0 wild or (JS only) x wild. Someone then chooses Powers of the Base even though no 1 cube is in Resources. Is this second selection legal?
- Ruling Yes, it is. Since 0 (or x) can be used for 1 to obtain Powers of the Base, the second selection has the potential to affect the shake. If the same player chooses both variations, the order in which they are circled makes no difference.
- A27** Situation (MJS only) One player selects Base Eight, then another chooses Black Exponent. However, the only digits on the black cubes in Resources are 8s and 9s.
- Ruling The player choosing Black Exponent is penalized a point and must select another variation because the first choice has no chance of affecting the shake. Even with 0 or x wild, neither of those symbols is on the black cubes.
- Penalty -1 for the player selecting Black Exponent
- A28** Situation (JS only) One player selects Number of Factors, and someone chooses x wild. Are these variations in conflict?
- Ruling No, they are not. x may be used for Number of Factors whether it is wild or not. Remember, however, that all x's in the Goal and Solution must represent the same symbol. If x is used as x, one x could be Number of Factors and another could be multiplication.
- A29** Situation (S only) No – signs are rolled. May a player select the Imaginary variation?
- Ruling Yes. The Imaginary variation affects the shake by allowing all roots of any number, real and complex.
- A30** Situation (S only) Imaginary and 0 or x wild are called. May a wild 0 or x be used as a sideways – to equal  $i$ ?
- Ruling Yes. The Equation-writer must specify, as usual, that the wild cube represents  $i$ .
- A31** Situation (S only) One player selects 0 or x wild. Then that player or an opponent chooses Log even though no  $\div$  cube is in Resources. Is this a legitimate selection?
- Ruling Yes, it is. Since the wild cube may be used for  $\div$ , the second variation has the potential to affect the shake. If the same player chooses both variations, the order in which they are circled makes no difference.

## Section B: Setting the Goal

### Part I: Situations Involving Only the Basic Rules

**NOTE** The following situations also apply to On-Sets (with appropriate modifications in some cases): **B1-6, 8-13, 23-24, 26-36.**

- B1** Situation A player other than the Goal-setter rolls the cubes.  
Ruling Illegal procedure; if the rightful Goal-setter chooses a variation after the cubes are rolled, the illegal procedure is insulated and the cubes are used as rolled (with the correct Goal-setter setting the Goal). However, if the illegal procedure has not been insulated, the rightful Goal-setter should reroll the cubes. (The time for doing this should be reset to one minute.)
- B2** Situation The Goal-setter rolls the cubes. Then another player sets the Goal.  
Ruling Illegal procedure; if illegal procedure is charged, the cube(s) in the Goal section of the mat (and any bonus cube in Forbidden) are returned to Resources and the rightful person sets the Goal. If the Goal-setter was being timed, the time should be reset to one minute after the interruption. If no one charges illegal procedure and a valid move or challenge is made, the Goal stands as set even though the wrong player set it. (This does not affect who is scheduled to be the Goal-setter for the next shake.)  
Comment If there is evidence of collusion between the player who rolled the cubes and the one who illegally set the Goal, apply the Behavior Rule.
- B3** Situation The Goal-setter calls "Bonus" and plays *two* cubes to Forbidden.  
Ruling Illegal procedure; the second cube played to Forbidden is returned to Resources or, if both cubes were played simultaneously, the Goal-setter returns either one of them to Resources.  
Penalty None unless (a) the Goal-setter's time expires before the illegal procedure is corrected and the Goal is set or (b) the Goal-setter is leading in the match and not allowed to make Bonus moves, in which case the Goal-setter is penalized one point. In the latter case, *both* cubes in Forbidden are returned to Resources.
- B4** Situation The Goal-setter places one or more cubes in the Goal section of the mat, then tries to return one or more of them to Resources.  
Ruling Illegal procedure; once a cube touches the mat in the Goal section, it must be used in the Goal. All the Goal-setter may do is arrange and group the cubes to form the Goal.  
Penalty None unless the Goal-setter's time expires  
Comment Players should be taught to form the Goal off the mat; then, after checking it, transfer the cubes to the mat.
- B5** Situation The Goal-setter places cubes on the Goal section of the mat, then returns one or more of them to Resources before completing the Goal. Before anyone charges illegal procedure, another player moves.  
Ruling Assuming the player who moved was scheduled to move first after the Goal-setter, the move is valid and insulates the illegal procedure so that the Goal stands as set. But if the player who moved was not scheduled to move first after the Goal-setter, the move is illegal procedure also and does not insulate the first illegal procedure. Now if someone charges illegal procedure, both the latest move and the cubes illegally in the Goal are returned to Resources.

- The Goal-setter then uses the cubes that originally touched the mat in the Goal.
- Penalty None unless the Goal-setter's time expires before the Goal is completed
- B6** Situation Same as **B5** but, before anyone charges illegal procedure or the next player moves, a player challenges Impossible against the Goal.
- Ruling The challenge insulates the illegal procedure and is worked out in the usual way. The Goal stands as set when the challenge is made.
- B7** Situation After moving some cubes to the Goal section of the mat, the Goal-setter returns one or more of them to Resources, then tries to complete the Goal with other cubes. After an opponent charges illegal procedure, the Goal-setter discovers that the original cubes cannot be arranged to form a legal Goal. For example, the cubes in the Goal are 2+3+\*–.
- Ruling The cubes played to the mat must remain on the mat. If the Goal-setter cannot figure out a way to arrange and group the cubes to form a legal Goal, then an opponent should challenge Impossible.
- Penalty -1 if the Goal-setter exceeds his time limit.
- B8** Situation The Goal-setter tries to change the Goal after it is set.
- Ruling If the Goal-setter said "Goal" or was asked if he was finished and said "yes," or started timing the next player or let the timing of the next player begin, then the Goal stands as set. If not, the Goal-setter may change the Goal by rearranging or regrouping the cubes in the Goal section or by adding one or more cubes to them (up to a limit of six cubes in Equations). However, the cubes already on the Goal section of the mat must stay in the Goal.
- Penalty None unless the Goal-setter's time expires before the Goal is finished
- B9** Situation The Goal-setter moves a cube to Required or Permitted before setting the Goal.
- Ruling Illegal procedure; the cube in Required or Permitted is returned to Resources. If a Goal was set, it stands and play continues.
- Penalty None unless the Goal-setter's time expires before the illegal procedure is corrected and the Goal is set.
- B10** Situation The Goal-setter is in process of placing the cubes of the Goal on the mat when the two minutes for setting the Goal expires.
- Ruling The Goal-setter must complete the Goal during the time limit.
- Penalty -1 for the Goal-setter (assuming a ten-second countdown occurred)
- B11** Situation The Goal-setter begins to set the Goal (that is, places one or more cubes on the Goal section of the mat), calls "Bonus" and moves a cube to Forbidden, then resumes setting the Goal.
- Ruling Illegal procedure; the cube in Forbidden is returned to Resources. As soon as the first cube of the Goal touches the mat, no bonus move may be made.
- Penalty None unless the Goal-setter's time expires before the Goal is finished
- B12** Situation Same as **B11** except that, in the process of setting the Goal, the Goal-setter moves a cube to Required or Permitted.
- Ruling Illegal procedure; the cube in Required or Permitted is returned to Resources.
- Penalty Same as **B11**

- B13** Situation Same as **B11** (or **B12**) except that, before anyone charges illegal procedure, the Goal is completed and someone challenges Impossible against the Goal.
- Ruling The challenge insulates the illegal procedure. The challenge stands and is worked out in the usual way. The cube moved to Required, Permitted, or Forbidden stays where it was played.
- Comment If the opponent challenges Now, the challenge is illegal if no cube is in Required or Permitted. Also, make sure there is no collusion between the Goal-setter and the Challenger.
- B14** Situation The Goal-setter sets a Goal of more than six cubes. Before anyone challenges the Goal, an opponent charges illegal procedure.
- Ruling The Goal is an illegal procedure. The extra cube(s) must be returned to Resources, and the Goal-setter must use just six of the cubes on the Goal section (the first six played unless they were all played simultaneously, in which case the Goal-setter may choose which six to keep in the Goal).
- Penalty None unless the Goal-setter's time expires
- Comment An opponent should have challenged Impossible since a Goal of more than six cubes has no defined interpretation in Equations. (See **B16** below.) However, if illegal procedure was charged first, the Goal-setter may change the Goal.
- B15** Situation Same as **B14** but no one charges illegal procedure, and the next player moves.
- Ruling The move insulates the illegal procedure, and the Goal stands as played.
- Comment An Impossible challenge should be issued against the latest Mover since the Goal has no defined interpretation in Equations.
- B16** Situation Same as **B14** but before anyone charges illegal procedure, a player challenges Impossible.
- Ruling The challenge insulates the illegal procedure. The Goal stands as played and the challenge is worked out in the usual way. Since a Goal of more than six cubes is not allowed in Equations, the Challenger is correct. However, the official should not tell players this until the Third Party has taken a side, and the players ask about the correctness of the challenge.
- B17** Situation The Goal-setter puts one or more cubes on the Goal section of the mat but has not said "Goal" or "Goal set" and the time was setting the Goal has not run out. An opponent picks up the Challenge block and challenges Never.
- Ruling Illegal challenge.
- Penalty -1 for the player making the illegal challenge.
- B18** Situation The Goal-setter sets a Goal containing a three-digit numeral.
- Ruling The Goal is not a legal expression and an opponent should challenge Impossible. If illegal procedure is called before anyone challenges the Goal and before a subsequent move is made, the Goal-setter must rearrange the cubes in the Goal and, if necessary, add more cubes to it (up to a total of six) until the Goal is a valid expression.
- Penalty If a challenge is issued, follow the rule for scoring after a challenge. Otherwise, no penalty unless the Goal-setter's time expires before a legal Goal is complete.



Comment (**MJS** only) If the Goal has an interpretation without a three-digit numeral (for example, with the Exponent variation in force for the third digit), the Goal is legal. (**S** only) Three consecutive digits could also be legal with 0 or x wild and Imaginary.

- B19** Situation Same as **B18** but before anyone charges illegal procedure, the next player moves.  
Ruling The move insulates the illegal procedure and the Goal stands as played. If the exception listed in the Comment for **B18** does not apply, the Goal is still undefined. Any Impossible challenge issued now is directed against the last Mover.
- B20** Situation The Goal-setter sets a Goal like  $-23$  (improper use of the  $-$  sign) or  $+19$  (illegal use of the  $+$  sign). That is, the Goal is a legal *mathematical* expression but not a legal Equations expression.  
Ruling Same as **B18**  
Penalty Same as **B18**
- B21** Situation Same as **B20** except that no one charges illegal procedure or challenges the Goal and one or more subsequent moves are made.  
Ruling The first move after the Goal insulates the Goal as an illegal procedure. However, the Goal is still not a legal expression, and an Impossible challenge should be made against the latest Mover.  
Penalty Same as **B18**
- B22** Situation The Goal-setter states orally how the cubes of the Goal are to be grouped but does not physically group them this way on the mat.  
Ruling The oral declaration is not binding, and players may interpret the Goal in any legal way.
- B23** Situation The Goal-setter sets a Goal like 06 or  $7 \times 06$ , etc.  
Ruling  $06 = 6$ . The digit before the 6 is not significant but may be used if the Goal-setter so desires.  
Comment The 0 would be significant if certain variations were in effect: 0 wild, Exponent (**MJS** only), or Decimal in Goal (**S** only).
- B24** Situation The Goal-setter places cubes on the mat but does not say “Goal,” and time expires.  
Ruling The only way the Goal-setter may add to, rearrange, or regroup the Goal is by paying a one-point penalty to gain an additional minute. Since, in most instances, the Goal-setter would not want to do this, the presumption is that the Goal is complete as it stands when the time expires.  
Penalty None unless the Goal-setter does wish an additional minute.
- B25** Situation **The Goal-setter places cubes on the mat so that the Goal extends off the right edge of the playing mat. Is any cube that is off the edge of the mat considered to have been played to the Goal and thus may not be returned to Resources?**  
Ruling **Yes. If the cube is clearly a continuation of the Goal, it is part of the Goal and may not be returned to Resources. If the Goal-setter has not said “Goal,” she may rearrange or regroup the cubes but may not return any of them to Resources.**

- B26** Situation The Goal-setter makes a bonus move, then commits an illegal procedure in setting the Goal. The Goal is charged as an illegal procedure. Is the cube in Forbidden returned to Resources?
- Ruling No. That move was not illegal procedure (assuming the Goal-setter is not leading in the match). Only the Goal is changed to correct the illegal procedure.
- Penalty None unless the Goal-setter's time expires before the Goal is finished or the Goal-setter is leading in the match.
- B27** Situation The Goal-setter, who is leading in the match, makes a bonus move, then sets the Goal. An opponent charges illegal procedure against the bonus move.
- Ruling Since you may not insulate your own illegal procedure, the Goal-setter loses a point and must return the cube in Forbidden to Resources.
- Penalty None unless the Goal-setter's time expires before the illegal move is returned to Forbidden.
- B28** Situation The Goal-setter commits an illegal procedure while setting the Goal, and the time expires. A one-point penalty is enforced, and the Goal-setter is given another minute. At the end of the additional minute, a legitimate Goal still has not been set.
- Ruling The Goal-setter loses an additional point and his turn. Any cubes on the Goal section and any bonus move in Forbidden are returned to Resources. (That is, the Goal-setter's entire turn is wiped away.) Setting the Goal passes to the player to the left of the previous Goal-setter. This player now has two minutes to set a Goal. The cubes are not rerolled. For the next shake, this second Goal-setter will roll the cubes and set the Goal as originally scheduled.
- Penalty The original Goal-setter loses two points and his turn.
- B29** Situation The Goal-setter sets a Goal, then picks up the challenge block and says "Goal."
- Ruling A player who picks up the challenge block must make a valid challenge. However, in this case, the Goal-setter is challenging himself, which is illegal. Therefore, picking up the challenge block is set aside and the shake continues.
- Penalty The Goal-setter is penalized one point for challenging himself.
- B30** Situation The Goal-setter sets a Goal, then, before the next player moves, picks up the challenge block and issues a challenge.
- Ruling Same as **B29**
- Penalty Same as **B29**
- B31** Situation The Goal-setter sets a Goal, then *touches* the challenge block and says "Goal."
- Ruling Set aside the touching of the challenge block. Tell the Goal-setter not to touch the block to call "Goal." The shake continues.
- Penalty None
- B32** Situation A player issues a Now challenge against the Goal when no cubes are in Required or Permitted.
- Ruling This challenge is illegal.
- Penalty The player attempting to challenge loses one point.

- B33** Situation The Goal-setter declares “no Goal.” The one-minute time is set for opponents to agree or disagree with the declaration. Before the end of the minute, an opponent disagrees. How much time does this player have to set a Goal?  
 Ruling The player disagreeing with the no Goal declaration has the rest of the one minute plus *one* additional minute to set the Goal.
- B34** Situation The Goal-setter puts one or more cubes on the Goal section of the mat, then removes the cube(s) and says “no Goal.”  
 Ruling Illegal procedure; once the first cube touches the mat, the Goal-setter is committed to setting a Goal. The “no Goal” declaration therefore has no effect; the cubes are returned to the mat and the Goal-setter must complete the Goal or let it stand as it was.  
 Penalty None unless the Goal-setter’s time expires before the Goal is finished.
- B35** Situation Same as **B34**; however, before anyone charges illegal procedure against the “no Goal” declaration, a player disagrees with the declaration.  
 Ruling The illegal procedure is insulated. The player disagreeing with the no Goal declaration sets a Goal, and the shake continues.  
 Penalty If an Impossible challenge is not successfully made against the Goal right after it is set, the original Goal-setter loses two points.
- B36** Situation Before putting any cubes on the Goal section, the Goal-setter calls “Bonus” and moves a cube to Forbidden. An opponent, believing that the variations and/or the Resources have created a no Goal situation, challenges Impossible.  
 Ruling Set aside the challenge. While the Bonus move implied that the Goal-setter thinks a makeable Goal can be set, no challenge can be issued before the Goal is set.  
 Comment The premature challenger is penalized one point.
- B37** Situation Same as **B36** except that after making the bonus move, the Goal-setter announces “no Goal.” Now one of the other players challenges Impossible.  
 Ruling Same as **B36**; the bonus move committed the Goal-setter to setting a Goal. The Goal-setter must set a Goal. However, no challenge can be issued until the Goal is finished.  
 Comment The premature challenger is penalized one point.
- B38** Situation Same as **B36** except that, after making the bonus move, the Goal-setter announces “no Goal.” One of the other players disagrees with the no Goal declaration.  
 Ruling The Goal-setter’s illegal procedure of declaring No Goal after making a Bonus move is insulated by the opponent’s disagreement with the declaration. The opponent must set a Goal.  
 Penalty If no one successfully challenges Impossible against the Goal right after it is set, the original Goal-setter loses two points.
- B39** Situation The Goal is  $3\sqrt{\sqrt{64}}$ . Is this legal?  
 Ruling Yes, it is. This Goal must be interpreted as  $3\sqrt{(\sqrt{64})}$ , that is, the cube root of the square root of 64, which equals  $3\sqrt{8}$  or 2.  
 Comment (**E** only)  $3\sqrt{\sqrt{64}}$  is a legal Goal in Elementary since each part equals a whole number. However, a Goal like  $3\sqrt{\sqrt{4}}$  is *not* valid in Elementary since it equals  $3\sqrt{(\sqrt{4})} = 3\sqrt{2}$ , which is not allowed since it does not equal a whole number.

- B40 Situation** The Goal-setter calls “No Goal.” What happens now?
- Ruling** Opponents must either agree with the No Goal declaration or challenge it.
- If all players agree, that shake is void, and the same player repeats as Goal-setter for a new shake.
  - An opponent who does not agree with the “No Goal” declaration indicates disagreement by picking up the challenge block and challenging the “No Goal” declaration. She then has two minutes to write a correct Equation (Solution and Goal). If there is a third player, he may also write an Equation. The Challenger and Third Party may use as many cubes from Resources as needed for the Goal and Solution.

**Comment** Scoring for this Challenge is as follows:

- If the Challenger presents a correct Equation, he scores 6. If the Challenger’s Equation is incorrect, he scores 2.
- If the Third Party presents an incorrect Equation, she scores 2. If the Third Party presents a correct Equation, she scores 4 if the Challenger’s Equation is also correct or 6 if the Challenger’s Equation is incorrect. If the Third Party does not present an Equation, she scores 6 if the Challenger’s Equation is incorrect or 2 if the Challenger’s Equation is correct.
- If either the Challenger or the Third Party presents a correct Equation, the original Goal-setter scores 2. If neither the Challenger nor the Third Party presents a correct Equation, the original Goal-setter scores 6.

**Section B: Setting the Goal**  
**Part II: Situations Involving Variations**

- B38** Situation The Goal-setter sets the Goal before variations are selected. Another player challenges Impossible against the Goal.  
Ruling The challenge insulates the illegal procedure of setting the Goal before choosing variations. Work out the challenge in the usual way. No variations are in play for any Solutions.  
Comment A Now challenge in this situation is illegal because no cubes are in Required or Permitted yet. So that challenge does not insulate the Goal-setter's illegal procedure.
- B39** Situation The Goal contains an expression that is allowed by a certain variation. However, that variation is not in effect for the shake. Examples: The goal is  $x13$  when Number of Factors was not chosen. (**EM** only) The Goal contains an upside-down cube but the upside-down variation was not selected.  
Ruling An opponent should challenge Impossible against the Goal because it is undefined. However, if an opponent charges illegal procedure before anyone challenges, the illegal procedure charge takes precedence, and the Goal-setter must rearrange the cubes in the Goal to make it a legitimate expression. (The cubes already played to the mat must stay there.)  
Penalty If illegal procedure is charged, the Goal-setter loses a point if he cannot complete an acceptable Goal before the two-minute time limit expires.  
Comment The official should not tell the players that the Goal should have been challenged unless one of them asks. If the Goal is challenged, the official should not tell players that it is impossible until the Third Party, if there is one, has taken a side, and players ask about the correctness of the challenge.
- B40** Variation Sideways  
Situation In the Goal, the Goal-setter puts a sideways cube immediately behind or in front of another digit.  
Ruling Same as **B39**  
Penalty Same as **B39**  
Comment (**MJS** only) This assumes the Exponent variation is not in force.
- B41** Variation Sideways  
Situation Same as **B40** except that no one challenges the Goal or charges illegal procedure, and one or more subsequent moves are made.  
Ruling An Impossible challenge should be made against the latest Mover.
- B42** Variation Sideways  
Situation In the Goal, may two consecutive digit cubes be turned sideways to give the reciprocal of a two-digit number?  
Ruling No; each numeral cube individually must be turned sideways, and an operation sign is required between the two digits. A Goal containing two consecutive sideways digits is undefined and falls under the ruling for **B39**.  
Comment **MJS**: This assumes no variation like Exponent is in force that would allow two consecutive sideways cubes.  
Penalty Same as **B39**

- B43** Variations Sideways, upside-down  
 Situation The Goal-setter claims a cube in the Goal is both sideways and upside-down.  
 Ruling Illegal procedure; it cannot be both. The physical placement of the cube by the Goal-setter determines whether it is sideways, upside-down, or right-side up. If the Goal-setter tried to place the cube on an angle (diagonal) so that it is neither sideways, upside-down, nor right-side up, the Goal is undefined. If an opponent charges illegal procedure before anyone challenges, the Goal-setter must reposition the cube in one of the three acceptable ways.  
 Penalty Same as **B39**  
 Comment A cube may be used both sideways and upside-down in the Solution when both variations are in play.
- B44** Variations Sideways, 0 Wild  
 Situation In the Goal, may 0 be turned sideways to give the reciprocal of a number?  
 Ruling Yes, it may. A player presenting a Solution may interpret a sideways 0 that is not part of a two-digit number in the Goal as 1, 1/2, 1/3, ..., 1/9 (but not 0 since 1/0 is undefined).  
 Comment Since the digit 0 on the cubes is longer vertically than it is wide, it is possible to determine whether a 0 in the Goal is right-side up or sideways.
- B45** Variation Upside-down  
 Situation In the Goal, may two consecutive digit cubes be turned upside-down to give the additive inverse of a two-digit number?  
 Ruling No; each numeral cube individually must be turned upside-down, and an operation sign is required between the two digits. A Goal containing two consecutive upside-down digits is undefined and falls under the ruling for **B39**.  
 Comment **MJS**: This assumes no variation like Exponent is in force that would allow two consecutive sideways cubes.  
 Penalty Same as **B39**
- B46** Variation Upside-down  
 Situation In the Goal, the Goal-setter puts an upside-down cube immediately behind or in front of another digit.  
 Ruling The Goal is undefined. Follow the ruling for **B39**.  
 Penalty Same as **B39**  
 Comment (**MJS** only) This ruling assumes no variation like Exponent is in force that would allow two consecutive sideways cubes.
- B47** Variation Upside-down  
 Situation Which way does a Goal face on the mat? For example, a Goal of  $7 + \nabla$  to a player on one side appears to be  $4 + \angle$  to a player on the opposite side of the mat.  
 Ruling The bottom of the Goal rests on the line segment on the mat with the word "GOAL" beneath it. Thus, if the Goal is placed on the mat as  $\underline{7 + \nabla}$ , it must be interpreted as  $7+(-4)$ .
- B48** Variation Upside-down  
 Situation A Goal is set on the mat like this.  
 Ruling The Goal is undefined. See the ruling for **B39** of this section.  
 Penalty Same as **B39** of this section

$$\begin{array}{c} \bar{6} \nabla 2 \\ \hline \text{GOAL} \end{array}$$

- B49** Variations Upside-down, 0 Wild  
 Situation The Goal-setter places a 0 in the Goal. May this 0 be interpreted right-side up or upside-down?  
 Ruling Physically there is no way to tell whether a 0 cube is right-side up or upside-down. Therefore, an Equation-writer may interpret the 0 either way (but must clearly indicate when writing his interpretation of the Goal what the 0 represents). The Goal-setter may not force any interpretation. Thus the 0 in the Goal (assuming it is not part of a two-digit numeral) may equal 0, 1, -1, 2, -2, . . . , 9, or -9.  
 Comment **(MJS)** If Base  $m$  is also in force, 0 may equal 0,  $\pm 1$ ,  $\pm 2$ , . . . ,  $\pm(m-1)$ .
- B50** Variations Upside-down, Average  
 Situation The Goal is  $6-\nabla$ . May this be interpreted as the average of 6 and 4?  
 Ruling No; the + must be explicitly used to obtain average. This Goal equals 10.
- B51** Variation 0 Wild  
 Situation The Goal-setter puts a 0 in the Goal and declares that it is to be interpreted as a 5 (or any other digit) or **(MJS only)** a particular operation.  
 Ruling The Goal-setter's "declaration" has no effect and should be ignored. An Equation-writer may interpret the 0 in the Goal as any numeral on the cubes or **(MJS only)** any operation if its placement in the Goal allows it.  
 Comment **(JS only)** The same ruling applies to x Wild.
- B52** Variation Number of Factors or **(E only)** Smallest Prime  
 Situation The Goal-setter places an x in the Goal and declares that it is to be interpreted as the Number of Factors (or Smallest Prime) operator.  
 Ruling The Goal-setter's "declaration" has no effect and should be ignored. The placement of the x within the Goal determines whether it means Number of Factors (or Smallest Prime) or multiplication. For example, in a Goal like  $x3^5$ , the x means Number of Factors (or Smallest Prime). However, the x in  $34x5$  represents multiplication.
- B53** Variations**(EM only)** Decimal Point, Factorial  
 Situation The Goal is  $4^3$ . May the Goal be interpreted as  $4.(3!)$  to give 4.6?  
 Ruling No, it may not.
- B54** Variations**(EM only)** Decimal Point, Factorial  
 Situation The Goal is  $6^{\wedge}$ . May the Goal be interpreted as  $(6!)^{\wedge}$ .  
 Ruling No, it may not.  
 Comment The Goal may be interpreted as  $(6^{\wedge})!$
- B55** Variations **(MJS only)** Sideways, Powers of the Base  
 Situation May a 1 cube be used sideways in the Goal to give the reciprocal of a power of the base?  
 Ruling Yes, it may. However, negative Powers of the Base are built into the variation already so that using the 1 sideways is redundant (but legal).
- B56** Variations **(MJS only)** Sideways, Exponent  
 Situation Suppose the Goal-setter sets a Goal like  $4\curvearrowright$  (where the 2 is the Exponent color) or  $\curvearrowright 2$  (2 the Exponent color). Are these legal?  
 Ruling Yes, they are.  $4\curvearrowright$  means  $4^{(1\div 2)}$ .  $\curvearrowright 2$  means  $(1\div 4)^2$ .

- B57** Variations **(MJS only)** Sideways, Base  $m$   
 Situation How is a sideways cube in the Goal to be interpreted?  
 Ruling The base does not affect the interpretation of the sideways cube as the reciprocal of the digit. A sideways 2 equals the reciprocal of 2 (one-half) regardless of the base. Thus  $(3 \times 4) \times \overline{2}$  is six regardless of the base.  
**(MJS only)**  $5 \wedge \overline{2}$  is  $\sqrt{5}$  regardless of the base.
- B58** Variations **(MJS only)** Upside-down, Powers of the Base  
 Situation May a 1 cube be used upside-down in the Goal and interpreted as the negative of a power of the base?  
 Ruling Yes, it may. This combination of variations allows a 1 cube to represent  $10^1$ ,  $-10^1$ ,  $10^2$ ,  $-10^2$ ,  $10^3$ ,  $-10^3$ , etc. (and similarly for another base if base  $m$  is chosen).
- B59** Variations **(MJS only)** Upside-down, Exponent  
 Situation Suppose the Goal-setter sets a Goal like  $4\overline{2}$  (red 2 where red is the Exponent color) or  $\overline{4}2$  (red 2). Are these legal?  
 Ruling Yes, they are.  $4\overline{2}$  means  $4 \wedge (-2)$ .  $\overline{4}2$  means  $(-4) \wedge 2$ .
- B60** Variation **(MJS only)** 0 Wild  
 Situation A Goal like 720 is set.  
 Ruling The only way this Goal is legal is if Exponent is chosen and the 0 is the proper color. Otherwise the Goal is undefined.  
 Comment **(JS only)** This ruling also applies to x Wild. **(S only)** The Goal would also be legal if  $\sqrt{=}i$  is in force because 0 may be  $i$ .
- B61** Variation **(MJS only)** 0 Wild  
 Situation The Goal is 702, 1702, 7012, or 17032.  
 Ruling In each case, the 0 must be interpreted as an operation sign, not a digit. The Goal may not contain a numeral with more than two digits. An exception would be if the Exponent variation is also called. For example, with Exponent, the 0 in 702 can be a digit if the 0 and the 2 are the Exponent color.  
 Comment **(JS only)** This ruling also applies to x Wild. **(S only)** Another exception to the ruling above occurs if Imaginary in force. The 0 in each Goal could be  $i$ .
- B62** Variation **(MJS only)** 0 Wild  
 Situation The Goal is 7020, 7200, or 0720.  
 Ruling Each of these Goals is undefined. In each case, the only way the Goal could make sense is by interpreting both 0's as digits, yet the Goal may not contain numerals with more than two digits. Follow the ruling of **B39**. An exception would occur if Exponent were in force. Then 7200 could be legal if both 0's were exponents.  
 Penalty Same as **B39**  
 Comment **(JS only)** This ruling also applies to x Wild. **(S only)** This ruling assumes  $\sqrt{=}i$  is not in force. If it were, the 0's in each Goal could be interpreted as  $i$ .
- B63** Variation **(MJS only)** 0 Wild  
 Situation The Goal is 73200, 73020, 70320, or 07320 or 732000, 730200, 730240, etc.  
 Ruling Each of these Goals is undefined for the same reason explained in **B62**. Follow the ruling of **B39**. The only exception would be if Exponent is in force and the last three or four digits of each Goal were the Exponent color.  
 Penalty Same as **B39**



Comment (**JS** only) This ruling also applies to x Wild. (**S** only) This assumes Imaginary is not in force. If it is, 73020 and 70320 would be legal with each 0 interpreted as  $i$ . The only way 73200 would be legal is if Exponent and  $\sqrt{=i}$  are chosen and the 2 is the Exponent color.

**B64** Variations (**MJS** only) 0 Wild, Powers of the Base

Situation May a 0 in the Goal be interpreted as an integral power of the base?

Ruling Yes, provided the 0 is not in a two-digit numeral. The Powers of the Base variation does not say the 1 *cube* may be Powers of the Base.

Comment (**JS** only) This ruling also applies to x Wild.

**B65** Variations (**MJS** only) 0 Wild, Exponent

Situation The Goal equals or contains an expression like 20 where the 0 is red (the chosen Exponent color). May the 0 represent, say, a 9, which is not on a red cube, but still be interpreted as an Exponent since the 0 is red?

Ruling Yes, it may.

Comment (**JS** only) This ruling also applies to x Wild when an x is the Exponent color.

**B66** Variations (**MJS** only) 0 Wild, Base Eight

Situation How may a 0 in the Goal be interpreted?

Ruling Because of Base Eight, the interpretation of the 0 as a digit in the Equation is limited to 0, 1, 2, 3, 4, 5, 6, 7.

Comment With 0 Wild and Base  $m$ , 0 may equal any digit from 0 through  $m-1$  (**JS** only: including  $\wedge$  for bases eleven and twelve and  $\sqrt{\quad}$  for Base Twelve). (**JS** only) This ruling also applies to x Wild.

**B67** Variations (**MJS** only) Number of Factors, Base Eight

Situation How is a Goal like x21 interpreted?

Ruling x21 means the Number of Factors of  $17_{\text{ten}}$ , which is 2. That is, convert any two-digit numeral to base ten first, then determine the Number of Factors of the number.

**B68** Variation (**MJS** only) Exponent

Situation The Goal-setter sets a Goal like 932 (red 2 where red is the Exponent color). Is this a legitimate Goal and, if so, how may it be interpreted?

Ruling 932 may not be interpreted as a three-digit numeral. The only legal interpretation is  $93^2$ .

Comment If the 3 is also red, the Goal could be interpreted as  $(9^3)^2$  but *not* as  $9^{(3^2)}$ .

**B69** Variation (**MJS** only) Exponent

Situation The Goal is  $4+12$  (red 2 where red is the Exponent color). How may this Goal be interpreted if the Goal-setter does not physically group the cubes?

Ruling The Goal may equal  $4+12$ ,  $4+(1^2)$ , or  $(4+1)^2$ .

**B70** Variation (**MJS** only) Exponent

Situation With red the Exponent color, the Goal-setter places a Goal on the mat like  $3^2$  where the red 2 is raised above the 3.

Ruling Illegal procedure; the 2 should be placed alongside the 3. The Goal-setter may not force the Exponent interpretation of the 2. The Goal may be interpreted as 32 or as  $3^2$ .

Penalty None unless the Goal-setter's time expires before the illegal procedure is corrected

- B71** Variation (MJS only) Exponent  
 Situation The Goal is 5232, where all digits but the 5 are red where red is the Exponent color. How may this Goal be interpreted?  
 Ruling The two correct interpretations are  $[(5^2)^3]^2$  and  $(52^3)^2$ .  $(5^2)^{(3^2)}$  and  $5^{\lfloor 2^{\lfloor 3^2 \rfloor} \rfloor}$  are *not* correct interpretations since the 3 is not taken as an Exponent by itself.
- B72** Variation (MJS only) Base Eight  
 Situation The Goal-setter places an 8 or a 9 cube in the Goal. An opponent challenges Impossible without waiting for the Goal-setter to complete the Goal.  
 Ruling The Goal will be undefined since the digits '8' and '9' do not exist in Base Eight. However, the Tournament Rule **VI-A** states: "... you may challenge another player who has just *completed* a move or set the Goal." So set aside the challenge until the Goal-setter completes the Goal.  
 Penalty -1 for the player challenging before the Goal is complete  
 Comment The same ruling applies to using a 9 cube in the Goal when Base Nine is chosen. If, before a challenge is made, illegal procedure is charged, set aside the illegal procedure since the Goal is not an illegal procedure.
- B73** Variations (JS only) x Wild  
 Situation If an x is in the Goal, may it be interpreted as a sideways or upside-down digit?  
 Ruling There is no physical way to tell whether an x cube is right-side up, upside-down, or sideways. However, the position of the x may limit its interpretation. For instance, in the Goal 2x3, the x must be an operation sign (unless Exponent is in force and the 3 is the proper color). In the Goal x+7, the x may be a sideways or upside-down numeral. However, in this case, the Goal-setter may not force an interpretation of the x and players may interpret it as they wish. In all cases, each Equation-writer must indicate in writing the interpretation of the x in the Goal.  
 Comment (S only) If Imaginary is also in effect, x could be interpreted as sideways –.
- B74** Variations (JS only) Base Eleven (or Twelve)  
 Situation The Goal-setter uses \* (or ^) in the Goal as a one-digit numeral; for example,  $^{\sqrt{2}}$ . When one or more Equations are presented, the Goal-setter claims the \* in the Goal is sideways (or upside-down) and therefore represents one-tenth (or  $-10$ ), not ten.  
 Ruling Players may interpret the \* (or ^) in the Goal as either right-side up, sideways, or upside-down. The Goal-setter's placement of the cube in the Goal is not significant.  
 Comment Careful examination of an \* cube reveals a minor difference in the lengths of the "spokes" of the asterisk so that a sideways \* could be distinguished from a right-side up \*. However, accepting such a distinction would force players to put every \* in the Goal under a microscope. Faded cubes complicate the problem. To avoid this unwanted controversy, the ruling above is made.  
 Comment Newer games have ^ on the cubes in place of \*. *To make the ^ act exactly like \*, players may not place a ^ cube sideways or upside-down in the Goal.* If a player does so, an Impossible challenge should be issued against the Goal.

- B75** Variations (**JS** only) 0 Wild, Base Eleven (or twelve)  
 Situation The Goal is  $0$ ,  $0+6$ ,  $12 \times 0$ , etc., with  $0$  as a one-digit numeral. May the  $0$  be interpreted as  $\wedge$  (ten) or (in Base Twelve) as  $\surd$  (eleven)?  
 Ruling Yes, it may. Of course, any  $0$  in a Solution must have the same meaning.  
 Comment This ruling also applies to  $x$  Wild.
- B76** Variations (**S** only) 0 or  $x$  Wild, Imaginary  
 Situation May a wild cube in the Goal be used for  $i$ ?  
 Ruling Yes, it may. By the Imaginary variation, the  $|$  is now a numeral, and the wild cube may represent any numeral (or operation) on the cubes.  
 Comment The  $0$  or  $x$  in the Goal may also mean  $-i$  since each cube is ambiguous regarding upside-down. If sideways is chosen, a sideways  $0$  or any  $x$  may mean  $1/i$  (since  $x$  is ambiguous regarding sideways).
- B77** Variations (**S** only) 0 Wild, Imaginary  
 Situation The Goal is  $203$ . Is this legal?  
 Ruling Yes, it is. The  $0$  must be interpreted as  $|$  or as an operation since a three-digit numeral is not allowed in the Goal. So two possible values of the Goal are  $\pm 6i$ .  
 Comment If Exponent is also chosen and the  $3$  is the Exponent color,  $203$  could be interpreted as  $20^3$  or as  $(\pm 2i)^3$  but not as  $2(i^3)$  since the  $2$  may not be placed in front of  $i^3$  without a  $x$  sign. This ruling and comment also apply to  $x$  Wild.
- B78** Variations (**S** only) Powers of the Base, Decimal in Goal  
 Situation A  $1$  cube is used as a single digit numeral in the Goal. May a Solution-writer assume a decimal point in front of the  $1$ ?  
 Ruling Yes; however, such an action is redundant since Powers of the Base allows the  $1$  to stand for negative Powers of the Base (which includes  $.1$  as  $10^{-1}$ ).
- B79** Variations (**S** only) Exponent, Decimal in Goal  
 Situation The Goal is  $32$  where the  $2$  is the Exponent color. How may this Goal be interpreted?  
 Ruling The Goal may be interpreted as  $32$ ,  $3.2$ , or  $.32$  or as  $3^2$ ,  $.3^2$ ,  $3^{\cdot 2}$ , or  $.3^{\cdot 2}$ .
- B80** Variations (**S** only) Multiple of 6, Imaginary  
 Situation Suppose the Goal is  $2|$  or  $|2$  (which both equal either  $2i$  or  $-2i$ ). What Solutions would be correct?  
 Ruling Possible Solutions would equal  $\pm 2i \pm 6$ ,  $\pm 2i \pm 12$ ,  $\pm 2i \pm 18$ , etc.  
 Comment Adjust the Solutions for other values of  $k$ . For example, if  $k = 7$ , use  $\pm 2i \pm 7$ ,  $\pm 2i \pm 14$ , etc.
- B81** Variations (**S** only) Base Eight, Imaginary  
 Situation The Goal is  $23|$ . How is this interpreted?  
 Ruling Since the  $23$  is in Base Eight,  $23|$  means  $19_{\text{ten}}$  times  $\pm i$ . ( $i$  is  $i$  regardless of the base.)
- B82** Variations (**S** only) Imaginary  
 Situation May  $|$  be used upside-down in the Goal to give  $-i$ ?  
 Ruling Yes, it may.
- B83** Variations (**S** only) Imaginary  
 Situation May  $|$  be used sideways in the Goal for  $1/i$  (which equals  $-i$ )?  
 Ruling No, it may not. But nothing is lost mathematically because  $|$  may be used upside-down to give  $-i$ .

- B84** Variation (S only) Imaginary  
 Situation The Goal is  $7|2$ ,  $37|23$ ,  $3|4|5$ , or  $9||6$ , and so on. Are these Goals legal?  
 Ruling Yes since the Imaginary variation allows a numeral to be placed before or after  $|$  without an explicit multiplication sign.  $7|2 = \pm 14i$ ;  $37|23 = \pm 851i$ ,  $3|4|5 = \pm 60i^2 = \pm 60$ ,  $9||6 = \pm 54i^2 = \pm 54$ .  
 Comment In the sample Goals above, any or all of the single-digit numerals could be upside down or sideways.
- B85** Variations (S only) Imaginary, Decimal in Goal  
 Situation May a decimal be placed in front of a  $|$  in the Goal?  
 Ruling No, it may not. A decimal point may be placed only in front of a right-side up digit (0, 1, 2, ... 9 and \* (or ^) and  $\sqrt{\quad}$  in bases eleven and twelve).
- B86** Variations (S only) 0 or x Wild, Log  
 Situation Depending upon its placement, may a wild cube in the Goal be interpreted as the Log operation?  
 Ruling Yes; the Log variation says the  $\div$  sign (not the  $\div$  cube) may represent Log. Since there is no way to tell whether an x cube is right-side up or sideways, an x used as  $\div$  may be interpreted as either division or Log. However, a wild 0 cube must be explicitly placed sideways to give Log.
- B87** Variations (S only) Decimal in Goal  
 Situation The Goal contains a sideways 3. May a Solution-writer assume a decimal point in front of the sideways 3?  
 Ruling No. A decimal point may be placed only in front of a right-side up digit.
- B88** Variations (S only) Decimal in Goal  
 Situation The Goal contains an upside-down 7. May a player, for a Solution, put a decimal point in front of the upside-down 7?  
 Ruling No. A decimal point may be placed only in front of a right-side up digit.
- B89** Variation (S only) Decimal in Goal  
 Situation The Goal is 12. May this be interpreted as .012, .0012, etc.?  
 Ruling No; the only legitimate interpretations are 12, 1.2, or .12...
- B90** Variation (S only) Decimal in Goal  
 Situation **The Goal is  $3\sqrt{125}$ . In her Equation, a player writes the Goal as  $3\sqrt{125}$  with no decimal point in front of 125, between two digits, or after the 5. The Equation-writer says that the decimal point is understood behind the 5.**  
 Ruling **The Equation is incorrect. The decimal point is not understood to be behind the 5 and must be explicitly written in the Equation.**

## Section C: Moving

### Part I: Situations Involving Only the Basic Rules

*NOTE* All situations in this section apply to On-Sets as well.

- C1** Situation A player moves out of turn.  
Ruling Illegal procedure; return the cube(s) to Resources and continue the shake in the proper order. After the interruption, the time of the player whose turn it really is should be reset to the full amount.  
Comment If the player who moves out of turn is leading in the match and makes a bonus move, then he has committed two illegal procedures. An opponent may charge either one. If the opponent charges illegal procedure against the bonus move, the player loses a point even though he moved out of turn.
- C2** Situation A player moves out of turn. Before anyone charges illegal procedure, someone issues a legal challenge against the move.  
Ruling The challenge insulates the illegal procedure. Therefore, the challenge stands and is worked out in the usual way. The player who moved out of turn is the Mover.
- C3** Situation A player moves out of turn. No one charges illegal procedure or challenges. Another player moves.  
Ruling An illegal procedure is insulated by a legal action by another player. The question is whether the latest move (the one after the illegal procedure) is a “legal action.” If the player who made the latest move is the one whose turn was taken by the illegal procedure, then the move is legal and insulates the illegal procedure, which then stands as played. But if the latest move was by a player other than the one whose turn was taken by the illegal procedure, then it too is illegal procedure and does not insulate the previous illegal procedure. All cubes played by the two latest “Movers” are returned to Resources and play resumes with the player whose turn it should have been before the illegal procedures were committed. The original (correct) Mover’s time should be reset to the full amount.  
Comment If players have been moving in the wrong order (e.g., counterclockwise instead of clockwise) for several moves, a judge should simply tell them to start moving in the right order from that point on, leaving all previous moves as played. In some situations (such as when most of the cubes have been moved to the mat), players may prefer to complete the shake moving counterclockwise.
- C4** Situation A player tries to make a bonus move without first calling “Bonus.”  
Ruling Illegal procedure; the Mover must say the word “Bonus” before the first cube touches the mat in Forbidden. The Mover is not entitled to another move on that turn, and the second cube (if one was moved) is returned to Resources.
- C5** Situation Same as **C4** except that, before anyone charges illegal procedure, someone makes a legal challenge.  
Ruling The challenge insulates the illegal procedure and is worked out in the usual way. The player who made the illegal bonus move is the Mover. Any cube(s) moved by the latest Mover stay on the mat where they were played.

- C6** Situation A player calls “Bonus,” then moves to Required or Permitted.  
Ruling Illegal procedure; the Mover is not entitled to another move. The second cube (if one was played) is returned to Resources. Play continues.  
Penalty –1 if the Mover cannot rectify the illegal procedure before time expires
- C7** Situation A player calls “Bonus” and moves a cube to Required or Permitted, then moves a second cube to Required, Permitted, or Forbidden. Before anyone charges illegal procedure, a legal challenge is made.  
Ruling The challenge insulates the illegal procedure and is worked out in the usual way. (See the ruling for **C5**.)
- C8** Situation Same as **C7** but the challenge is issued before the second cube is played.  
Ruling Same as **C7**
- C9** Situation After moving a cube to one section of the playing mat, the Mover attempts to transfer the cube to another section.  
Ruling Illegal procedure; the cube must stay in the section where it first touched the mat. Play continues.  
Comment In On-Sets, an exception is allowed by the Shift from Permitted variation.
- C10** Situation So many cubes have been played to Required, Permitted, or Forbidden that they fill that area. The next player moves a cube to that section and places it atop another cube in that section. The Mover then tries to change the move, arguing that the first cube never “touched the mat.”  
Ruling Changing the move is illegal procedure. The play was complete when the cube touched the other cube(s) on the mat. Leave the cube where it was originally played.
- C11** Situation Same as **C10** but after the player changes the cube to another section of the mat, an opponent makes a legal challenge.  
Ruling The challenge insulates the illegal procedure. The cube stays in the *second* section to which it was moved. The challenge is worked out in the usual way.
- C12** Situation On his turn, a player moves two cubes to Required or Permitted or moves one cube to Required and one to Permitted.  
Ruling Illegal procedure; the Mover must return the second cube to Resources or, if both cubes were played simultaneously, choose one of the cubes to be returned. Play continues.  
Penalty Same as **C6**
- C13** Situation Same as **C12** except that, before anyone charges illegal procedure, an opponent makes a valid challenge.  
Ruling Same as **C7**
- C14** Situation A player moves a cube to the mat but sets it on the dividing line between two sections so that opponents are not sure where it belongs. Or, during the shake, a cube gets pushed onto the dividing line between two sections.  
Ruling One of the opponents must ask the Mover where the cube belongs. Players may not reason: “The Mover did not specify where the cube goes; therefore, I have the right to interpret it either way.” The Mover must specify which section the cube is in. If a cube gets pushed onto the dividing line during the shake, players must try to agree where it was played originally. If they cannot, an official may have to order the shake replayed.

Comment Sometimes, when Equations are presented, a player claims that a cube in one section of the mat was actually played to another section. Unless two of three players agree that the cube was played to the other section, the cube remains where it rests at the end of the shake.

**C15** Situation Same as **C14** but no one asks for a clarification of where the cube belongs, and one or more subsequent moves are made.

Ruling The player who made the ambiguous move should now be asked where the cube belongs.

**C16** Situation Same as **C14** but no one asks for a clarification of where the cube belongs. Eventually Equations are presented.

Ruling In order to check Equations, opponents will finally have to ask the player who made the ambiguous move to state where the cube belongs. Of course, if the Third Party has taken a stand (on a challenge) and Equation(s) have been presented without the position of the ambiguous cube being clarified, the opponents are giving the player who made the ambiguous move leeway to decide which section he wanted it in on the basis of the challenge that was made or the Equation(s) that have been presented. For example, if the cube sat on the borderline between Permitted and Forbidden and an opponent is using the cube in his Equation, the player who made the move can now say that he wanted it in Forbidden. The idea of this ruling and the ones in **C14** and **C15** is that the burden is on the opponents to demand clarification of where the Mover wants that cube. If they continue play without any clarification, they take a big risk.

Comment Sometimes players may not remember who moved the cube that sits on the dividing line. In this case, the benefit of the doubt should be given to the Equation-writer in interpreting where the cube was moved.

**C17** Situation A player calls “Bonus” and moves a cube to Forbidden. Before a second move is made, the Mover’s time expires. The Mover tries to “take back” the “Bonus” declaration in order to complete the turn within the time limit.

Ruling The Mover is committed to making a second move on this turn.

Penalty –1 for the Mover, who has one more minute to complete the turn

Comment If the Mover has said “Bonus” but has not moved to Forbidden when the ten-second countdown starts, the Mover could negate the “Bonus” declaration and complete the turn within the time limit by moving to Required or Permitted.

**C18** Situation The player in the lead calls “Bonus” and moves a cube to Forbidden. Before a second move is made, the Mover’s time expires. Does the mover receive two -1 penalties – one for not moving in time and the other for a bonus move when ahead

Ruling No. As soon as the first cube touched the mat in Forbidden, opponents should have penalized the mover a point for attempting to make a Bonus move when ahead. However, an additional penalty for the illegal procedure is that the mover may not make a second move. So any time limit penalty is set aside. Opponents may not delay calling the illegal procedure for the Bonus move in order to tag on a second -1 for the time violation.

Comment The comment for **C17** above applies here also.

- C19** Situation The player in first place in the match makes a bonus move.  
Ruling Illegal procedure; the second cube (if there was one) is returned to Resources.  
Penalty The Mover loses one point.
- C20** Situation Two or more players are tied for first in the match. May the tied players make bonus moves?  
Ruling Yes, they may.
- C21** Situation The player ahead in the match calls “Bonus” but then plays a cube to Required or Permitted.  
Ruling The player has committed one illegal procedure which negates another. Moving the first cube to Required or Permitted after calling “Bonus” is an illegal procedure. The penalty is that the player may not make a second move. However, the Mover has avoided being penalized for making a bonus move while ahead in the match.
- C22** Situation A Mover slides a Resource cube through one section of the mat in order to move it to another section. For example, the Mover slides a cube through Permitted to Forbidden.  
Ruling As soon as a Resource cube touches the mat in Required, Permitted, or Forbidden, it is played to that section. Therefore, the Mover must leave the cube in the first section that it touched.  
Comment If an opponent challenges as the cube is slid through one section on its way to another, the challenge stands because the cube is returned to the first section it touched. However, if a player challenges *after* the cube comes to rest in the second section it touched, the challenge insulates the illegal procedure and the cube stays where it came to rest. If it is not clear when the challenge was made, the Challenger may specify where he thought the cube was played when he challenged and it should be placed in that section before the Third Party takes a side and Equations are presented.
- C23** Situation A Mover takes a Resource cube and holds it just above the surface of one section of the mat before moving it to another section. In the meantime, an opponent challenges, thinking that the cube touched the section it hovered over.  
Ruling A cube is not played until it touches the mat in Required, Permitted, or Forbidden. So if the challenge was made before the cube touched the mat, the challenge is directed against the last move prior to the cube that was held just above the mat. If that move was made by the Challenger, the challenge is set aside since you may not challenge yourself. If the challenge was not made until the cube touched the mat, it is directed against the move in the section where it was actually placed even though this was not the Challenger’s intention. However, if there is evidence that the Mover intentionally held the cube close to one section before shifting it to another in order to “sucker” a challenge, the challenge can be set aside and the shake continued with the cube in the section where it was eventually placed. The Mover should be warned not to repeat this action on future moves.  
Penalty If the challenge was made before the cube touched the mat so that the Challenger is challenging himself, the Challenger is penalized one point. If the



Mover is warned against repeating the action but does so again later in the round, the Mover should be penalized one point.

- C24** Situation A player must leave the table during the round.  
Ruling The timer should be laid down until the player returns. The time that expires during the player's absence should be added to the time for the round at that table. When the player returns, play should continue where it left off.  
Comment A judge should stay at the table during the player's absence to make sure no cubes are moved and opponents do not look at the absent player's paper.
- C25** Situation One cube is left in Resources. The player whose turn it is moves the cube to Required or Permitted. An opponent charges illegal procedure.  
Ruling This is not illegal procedure. The move stands as played. Unless an opponent challenges Impossible, each player must write a Solution within two minutes.
- C26** Situation One cube is left in Resources. The player whose turn it is does not move the cube but instead says "forceout."  
Ruling The player's declaration has no effect, and it is still his turn. He must either challenge Impossible against the previous Mover or move the last cube to Required or Permitted.  
Penalty -1 if the player does not challenge Impossible or legally move the last cube before the one-minute time limit expires
- C27** Situation One cube is left in Resources. The player whose turn it is moves the cube to Forbidden. An opponent charges illegal procedure.  
Ruling This is illegal procedure. The last cube must be moved to either Required or Permitted.  
Penalty -1 if the Mover's time expires before he moves the cube to Required or Permitted (or challenges Impossible).
- C28** Situation Same as **C27** except that the opponent challenges Impossible within the first minute after the last cube was moved rather than calling illegal procedure.  
Ruling A special tournament rule covers this situation. The challenge is set aside and the mover must return the cube to Resources. The mover must either challenge Impossible or move the cube to Required or Permitted.  
Penalty None for the player challenging Impossible (unless the challenge occurred after the first minute following the illegal move); -1 for the Mover if his time expires before he moves the last cube to Required or Permitted or challenges Impossible  
Comment This situation is made an exception to the insulation rule to avoid collusion between two players against the third.
- C29** Situation A player charges illegal procedure against an opponent but does not clearly state the nature of the illegal procedure "immediately" (**IX-A** of the Equations Tournament Rules).  
Ruling The player must explain the illegal procedure to the judge. If the judge agrees with the illegal procedure charge, the opponent must take appropriate corrective action (for example, by returning a cube to Resources that was moved out of turn). If the illegal procedure charge is erroneous, the judge should warn the player that taking excessive time to make an illegal procedure charge will result in a -1 penalty in the future.

Penalty None unless the player had already been warned not to waste time making illegal procedure charges, in which case the player is penalized one point.

Comment If the delay occurred during the last five minutes of the round, the judge should add an appropriate number of minutes to the time for the table.

**Section C: Moving**  
**Part II: Situations Involving Variations**

- C30** Variation Sideways  
 Situation A Mover places a numeral cube sideways in Required or Permitted, claiming this placement of the cube means it must be used sideways in any Solution.  
 Ruling The Mover's placement of the cube is not binding. It may be used right-side up or sideways as each Equation-writer prefers.
- C31** Variation Upside-down  
 Situation A Mover places a numeral cube upside-down in Required or Permitted, claiming this placement of the cube means it must be used upside-down in any Solution.  
 Ruling The Mover's placement of the cube is not binding. It may be used upside-down or right-side up as each Equation-writer prefers.
- C32** Variation 0 Wild or (**JS** only) x Wild  
 Situation A Mover plays a wild cube to Required and states that it is to represent 7 (or any other numeral) or (**MJS** only) a particular operation.  
 Ruling The Mover's statement has no effect. An Equation-writer may use the wild cube for any numeral or (**MJS** only) any operation on the cubes.
- C33** Variation Number of Factors or (**E** only) Smallest Prime  
 Situation The Mover plays an x cube to Required and states that the x must be used as Number of Factors (or Smallest Prime) and not multiplication (or vice-versa).  
 Ruling The Mover's statement has no effect. Equation-writers may use the x as Number of Factors (or Smallest Prime) or multiplication.
- C34** Variation Multiple Operations  
 Situation A Mover places an operation cube in Required and states that the operation must be used more than once in any Solution.  
 Ruling The Mover's statement is not binding. Equation-writers may use the operation sign one or more times as they wish.
- C35** Variation (**E** only) LCM  
 Situation A Mover plays a  $\sqrt{\quad}$  cube to Required and states that it must be used for LCM and not root (or vice-versa).  
 Ruling The Mover's statement has no effect. Equation-writers may use the  $\sqrt{\quad}$  for LCM or root.  
 Comment The same ruling would apply to \* for GCF.
- C36** Variation (**E** only) Three-operation Solution but *not* Multiple Operations  
 Situation Two operation signs are in Required and/or Permitted. A Mover plays the last operation sign in Resources to Forbidden.  
 Ruling An opponent should challenge Impossible against the Mover since there is no way a Solution can now obey the variation. If an opponent charges illegal procedure, the judge should state that the move is not illegal procedure.

- C37** Variation (EM only) Percent  
 Situation A player moves a  $\sqrt{\quad}$  cube to Required and places it upside-down, stating that this means each player must use the cube as Percent.  
 Ruling The Mover's action has no effect. Equation-writers may use the  $\sqrt{\quad}$  for Percent or root.
- C38** Variation (EM only) Decimal Point  
 Situation A Mover plays an  $*$  cube to Required and states that the  $*$  must be used as a Decimal Point and not as the exponentiation operator (or vice-versa).  
 Ruling The Mover's statement has no effect. Equation-writers may use the  $*$  for Decimal Point or exponentiation.
- C39** Variation (EM only) Remainder  
 Situation A Mover plays a  $\div$  cube to Required and places it sideways, claiming that the cube must be used for Remainder.  
 Ruling The Mover's statement has no effect. Equation-writers may use the  $\div$  for Remainder or division.
- C40** Variation (MJS only) Powers of the base  
 Situation A player moves a 1 cube to Required or Permitted and states that the 1 must be interpreted as 10 (or some other specific power of the base).  
 Ruling The Mover's statement has no effect. Equation-writers may use the 1 for any power of the base.
- C41** Variation (MJS only) Exponent  
 Situation A player moves a numeral of the selected exponent color to Required or Permitted, placing it immediately behind another digit. The Mover claims that this placement of the cube means it must be used as an exponent for the other digit in any Solution.  
 Ruling The Mover's action has no effect. Equation-writers do not have to use the numeral as an exponent and, if they do, do not have to use it behind the other digit.
- C42** Variation (MJS only) Base Eight  
 Situation A player moves an 8 or 9 cube to Required.  
 Ruling An opponent should challenge Impossible since there is no digit "8" or "9" in Base Eight.  
 Comment The same ruling applies if Base Nine is in effect and a "9" is moved to Required.
- C43** Variation (MJS only) Base Eight  
 Situation Same as **C41** except that the 8 or 9 is moved to Permitted.  
 Ruling There is nothing wrong with the move. An Impossible challenge is worked out in the usual way. Equation-writers must ignore the 8 or 9 in Permitted.
- C44** Variation (MJS only) Base Eight  
 Situation Same as **C41** except that, before anyone challenges, an opponent charges illegal procedure.  
 Ruling The move is not illegal procedure. The illegal procedure charge is set aside and play continues.  
 Comment The judge must not tell players that an Impossible challenge should be made.

- C45** Variation (JS only) Base Eleven (or Twelve)  
Situation A player moves an \* cube to Required and states that it must be used as the digit ten and not for exponentiation (or vice-versa).  
Ruling The Mover's statement has no effect. Equation-writers may use the \* for the digit ten or exponentiation.  
Comment The same ruling applies to  $\sqrt{\quad}$  in Base Twelve and (S only) Log.
- C46** Variation (S only) Log  
Situation A player places a  $\div$  cube sideways in Required and states that it must be used as Log.  
Ruling The Mover's statement has no effect. Equation-writers may use the  $\div$  sign for division or Log.
- C47** Variation (S only) Imaginary  
Situation A player places a  $-$  cube sideways in Required and states that it must be used as  $i$ .  
Ruling The Mover's statement has no effect. Equation-writers may use  $-$  for subtraction or  $i$ .

## Section D: Challenging

*NOTE* All situations in this section also apply to *On-Sets*.

- D1** Situation After the Goal is set, one or more cubes are played to Forbidden but none to Required or Permitted. A player challenges Now.  
Ruling The challenge is illegal and is set aside.  
Penalty -1 for the player making the illegal challenge
- D2** Situation The Goal is set, and an opponent challenges Impossible before any additional move is made.  
Ruling The challenge is legal and is worked out in the usual way.
- D3** Situation Player *A* moves. Player *B* attempts to challenge player *C*.  
Ruling *B*'s challenge stands (assuming he picked up the challenge block) but is redirected at *A*, the last Mover.
- D4** Situation Player *A* moves. Before the next player makes a move, *A* picks up the block and states a challenge.  
Ruling You may not challenge yourself. Set *A*'s challenge aside. Play continues.  
Penalty -1 for *A*
- D5** Situation Player *A* calls "Bonus" and moves a cube to Forbidden, then attempts to challenge one of the other players.  
Ruling Since player *A* made the last move, *A* is challenging himself, which is illegal. The challenge is set aside and play continues.  
Penalty -1 for *A* and, if *A* were leading in the match, another -1 as well
- D6** Situation A player picks up the challenge block but refuses to state a challenge.  
Ruling The player's challenge is set aside.  
Penalty -1 for the false challenger
- D7** Situation A player picks up the challenge block but, before saying what he is challenging, wishes to retract the challenge.  
Ruling See the ruling for **D6**.  
Penalty -1 for the false challenger
- D8** Situation A player picks up the challenge block and says "Goal" or "illegal procedure" or (in *On-Sets*) "Universe."  
Ruling A player picking up the block must issue a valid challenge. If the player picking up the block was the last Mover, handle as in **D4** above. If the player is not the last Mover, handle as in **D7**.  
Penalty See **D4** and **D7** above.
- D9** Situation A player presents an Equation. Upon noticing an error in the Equation, an opponent "challenges" that player.  
Ruling Set aside the "challenge." The Equation must be accepted as correct or proved incorrect.  
Penalty -1 if the opponent picked up the challenge block

- D10** Situation After one player challenges, an opponent attempts to “counter-challenge” or make another “challenge.”
- Ruling Set aside the second “challenge.” Work out the first challenge in the usual way.
- Penalty The second player loses a point if he picked up the challenge block.
- D11** Situation A challenge is made. Before the Third Party indicates whether he will present an Equation, the Challenger or Mover shows an Equation.
- Ruling The Third Party must still be given an opportunity to decide whether to present an Equation. The time lost in settling the situation should be restored to the Third Party to make a decision.
- D12** Situation A player calls “Bonus” and moves a cube to Forbidden. Before the player makes his regular move, a challenge is made.
- Ruling The challenge stands. The move to Forbidden was a valid move and therefore the Mover may be challenged before the second move is made.
- D13** Situation A challenge and an illegal procedure charge are made at the same time; that is, one player picks up the block and another player charges illegal procedure at the same time.
- Ruling The challenge prevails. Set aside the illegal procedure charge and work out the challenge in the usual way.
- D14** Situation A player commits an illegal procedure, and an opponent charges illegal procedure. Another player then challenges this opponent that he should have challenged the move rather than charging illegal procedure.
- Ruling The illegal procedure charge takes precedence and may not be challenged. Set aside the “challenge.”
- Penalty –1 if the opponent picked up the block
- D15** Situation A player *touches* the challenge block and states a challenge. No one realizes until later that the player did not pick up the block.
- Ruling The illegal procedure of not picking up the block would be insulated by a valid action by another player, which would be either the Third Party stating whether she will present an Equation or one or more Equations being presented. If any of these actions occurred, the challenge stands as issued. However, if the Third Party has stated whether he will present an Equation and no Equations have been presented yet, the “challenge” is set aside until the same player (or the player who was the Third Party) picks up the block.
- D16** Situation A player touches the challenge block and says “Goal” or “illegal procedure” or (in On-Sets) “Universe.”
- Ruling Set aside the touching of the block but warn the player not to touch the block in any of these situations.
- Comment A player who frequently touches the challenge block or keeps a hand or finger on or close to it throughout the shake should be warned to stop. If the player persists, a one-point penalty should be enforced.

- D17** Situation A player picks up the challenge block and says “Challenge Win” or “Challenge Trap.”
- Ruling These are older *LinguiSHTIK* challenges and are not correct terminology for *Equations* and *On-Sets*. If an opponent asks for a clarification of the challenge, the Challenger must rephrase the challenge as “Now.”
- Comment If necessary, the judge may explain that the player is challenging Now.
- D18** Situation A player picks up the challenge block and says “Challenge Possible” (or “A Flub”) or “Challenge Never” (or “P Flub”).
- Ruling These are challenges from an earlier form of *Equations* and *On-Sets*. If an opponent asks for a clarification of the challenge, the Challenger must rephrase the challenge as “Now” or “Impossible”.
- Comment A judge may have to translate the Challenger’s language for the other players.
- D19** Situation A player reaches for the challenge block but knocks it aside without picking it up. The player then decides he doesn’t want to challenge.
- Ruling Since the player did not pick up the challenge block, he does not have to challenge. Play continues. (The Third Party could pick up the block and challenge at this point.)
- D20** Situation One cube remains in Resources or no cube remains in Resources. A player picks up the challenge block and says “Now.”
- Ruling A player may challenge Now only if there are at least *two* cubes in Resources. The challenge is illegal at this point in the shake and is set aside.
- Penalty -1 for the false challenger
- D21** Situation A player makes an illegal challenge. For example, with one cube left in Resources, a player challenges Now. Before the Mover objects to the illegal challenge, the Third Party indicates whether he will or will not present an Equation.
- Ruling The Third Party indicating whether he will present an Equation does not insulate the illegal challenge. Ignore the Third Party’s indication and set aside the challenge.
- Penalty The illegal challenge earns a -1 penalty.
- D22** Situation Same as **D21** but the Mover begins the time limit for writing Equations.
- Ruling The Mover has implicitly accepted the challenge as valid. So the challenge is worked out and scored in the usual fashion.
- Comment The same ruling applies if the Mover, without ever starting the time for writing Equations, accepts an Equation for checking.
- D23** Situation With three cubes left in Resources, the Mover calls “Bonus,” moves one of the cubes to Forbidden, then another to Required, Permitted, or Forbidden. An opponent challenges Now.
- Ruling Since only one cube remains in Resources after the Mover’s turn, the Now challenge is set aside. If no one challenges Impossible, the player whose turn it is moves the last Resource cube to the mat.
- Penalty –1 for the player challenging Now
- Comment An opponent may challenge the Mover after the Bonus move and before the regular move.



- D24** Situation No cubes remain in Resources. A player (other than the last Mover) picks up the challenge block and says “Now.”
- Ruling A Now challenge must be made with at least two cubes left in Resources. So this challenge is illegal and is set aside.
- Penalty –1 for the player challenging Now
- D25** Situation No cubes remain in Resources. A player (other than the last Mover) picks up the challenge block and says “Impossible.”
- Ruling The challenge is legal provided it was made before the end of the first minute of the two minutes for writing Equations after the last Resource cube has been moved to the mat. If it was made after the first minute expires, the challenge is set aside.
- Penalty If the challenge is set aside, the player who attempted to challenge loses a point.
- D26** Situation Same as **D25**. If the Impossible challenge was made within the first minute after the last Resource cube was moved, how much time do the Mover and the Third Party have to present an Equation?
- Ruling Equation-writers have one more minute after the first minute to complete their Equations.
- Comment In *On-Sets*, with the Two Solutions variation, players have *two* minutes after the Impossible challenge to complete Solutions.
- D27** Situation A player challenges Now and moves a cube from Resources to the mat.
- Ruling Illegal procedure; the cube is returned to Resources and the challenge is worked out the usual way.
- D28** Situation Same as **D27** except that no one charges illegal procedure and one or more Equations are presented.
- Ruling Moving the cube after challenging Now was illegal procedure. Also a player may not insulate his own illegal procedure. So if the challenger presents an Equation, but an opponent charges illegal procedure before starting to check the Equation, the cube moved after the challenge is returned to Resources. However, if opponents begin to check the Challenger’s Equation or the Third Party’s, the illegal procedure is insulated. The cube remains where it was moved. Equations may still use one *more* cube from Resources.
- D29** Situation Variations have been picked, but the Goal-setter has not finished setting the Goal. An opponent challenges Impossible.
- Ruling The Equations Tournament Rules state: “... you may challenge another player who has just *completed* a move or *set* the Goal.” So set aside the challenge until the Goal-setter completes the Goal.
- Penalty -1 for the false challenger
- D30** Situation After a challenge, the Third Party indicates whether she will present an Equation but then changes her mind.
- Ruling If the Third Party clearly indicated whether she is presenting an Equation, she cannot change the declaration. Examples of “clearly indicating” a decision are pointing to the player or saying “him” or “her,” stating “I will present an Equation,” answering “yes” when asked “Will you present an Equation?” etc.

- D31** Situation Two players grab the challenge block simultaneously. One ends up with a bigger piece of it or wrestles it away from the other player. Both players want to make the same challenge.
- Ruling Both players are equal as Challengers (assuming they are making the same challenge). Both can score 6 points if correct
- D32** Situation Two players grab the challenge block simultaneously. One challenges Now, but the other challenges Impossible.
- Ruling
- a. Have both the Now challenger and the Mover write Equations.
  - b. Check the Now challenger's Equation first. If it is correct, then that player scores 6 and the other two players score 2. The Impossible challenge has been proven wrong.
  - c. If the Now challenger's Equation is incorrect, that Challenger scores 2. Then check the Mover's Equation. If it is correct, the Mover scores 6 and the Impossible Challenger scores 2. If the Mover's Equation is incorrect, the Impossible Challenger scores 6, and the Mover scores 2.
- D33** Situation Same as **D32** except that one of the simultaneous challengers is the Mover.
- Ruling The situation resolves itself because the Mover's challenge is set aside since you cannot challenge yourself.
- Penalty The Mover is penalized one point for an illegal challenge.
- D34** Situation After a Challenge, the Mover or Challenger pushes the Third Party to "take a side" on the Challenge or in some way indicate whether the Third Party will present an Equation.
- Ruling This is unacceptable behavior by the Mover or Challenger since the Third Party has two minutes to decide whether to present an Equation. Warn the Mover or Challenger (or both) that repeating the behavior will result in a -1 penalty.
- Penalty None unless the Mover or Challenger was previously warned about the unacceptable behavior.

## Section E: Writing and Checking Equations

### Part I: Situations Involving Only the Basic Rules

**NOTE** All situations in this section also apply to *On-Sets* except **E8** and **E16-18**. For *On-Sets*, all references to “Equation” or “Equation-writer” should be interpreted as “Solution” or “Solution-writer” respectively. Also, references to two minutes to write or check Equations must be extended to three minutes when Two Solutions is played in Senior On-Sets.

- E1** **Situation** After presenting an Equation, a player attempts to change it or add to it.  
**Ruling** If the player (a) said “Equation,” or (b) was asked if he was finished and said yes, or (c) handed his paper to an opponent, then the Equation stands as presented even if the time has not expired. In the event that none of these conditions is met, the Equation-writer may change or add to the Equation.  
**Penalty** –1 if the player did not present the Equation and therefore may change or add to it but exceeds the time limit when doing so.  
**Comment** If an opponent *takes* the paper from the Equation-writer, this does not mean the writer was finished. However, if the writer takes back the paper after it was legally presented and makes a change, he is admitting that the original Equation was wrong. So the checker does not have to determine what the writer changed or added to the Equation. No further checking is necessary. The Equation is incorrect regardless of how the writer changed it.
- E2** **Situation** A player presents an Equation that is not circled.  
**Ruling** The Equation-writer must circle the Equation when an opponent asks him to do so.  
**Comment** If several Equations are circled on the paper and players are arguing over which Equation to check, the official should ask the Equation-writer to specify, in the official’s presence, which Equation is the desired one. If there has been confusion, the official can order the time for checking the Equation to be reset to two minutes.
- E3** **Situation** A checker claims one or both sides of the Equation are ambiguous.  
**Ruling** The checker should rewrite the Equation on his own paper and add symbols of grouping to show there is an interpretation for which the Solution does not equal the Goal. If the opponent succeeds in doing this, the Equation is incorrect. However, the opponent has only *one* chance to interpret the Equation. If that interpretation still equals the Goal, the player must accept the Equation. However, the other checker, if there is one, also has one opportunity to show there is an interpretation that makes the Equation incorrect but must do so within the same time period as the other checker.  
**Comment** The opponent may not add grouping symbols in a way that violates a grouping the Equation-writer has chosen. For example, suppose the Equation is this.

$$2 \times (3 + 5) \div 2 = 8$$

The opponent may *not* insert brackets like this:

$$2 \times (3 + [5] \div 2) \quad \text{or} \quad [2 \times (3)] + 5 \div 2.$$

**E4** Situation An Equation contains an odd number of parentheses and other symbols of grouping. That is, either an open parentheses (or bracket or brace) or a close parentheses (or bracket or brace) is missing.

Ruling The Equation is not automatically wrong. However, an opponent may copy the Equation onto his paper and add the missing grouping symbol in any appropriate place in order to create a grouping that makes the Equation not equal to any value of the Goal. If the opponent can do this, the Equation is incorrect. However, if there is no place the extra grouping symbol can be inserted to make the Equation not equal the Goal, the Equation is correct (assuming it satisfies all other requirements). Here are examples. Suppose the Equation for a Goal of 8 is presented like this:

$$2 + (3 \times 4 - 6.$$

An opponent may group the Equation as:

$$2 + (3 \times (4 - 6)) \text{ or } 2 + [(3) \times (4 - 6)]$$

By contrast, suppose the Equation is written like this:

$$2 + [(3 \times 4) - 6$$

In this case, there is no way a ] can be placed to make the Equation not equal 8. The checker may *not* group as

$$2 + [(3) \times [(4) - 6]]$$

because the grouping of  $3 \times 4$  has been violated.

Comment See the comment following **E3** on the previous page for limitations on where the extra grouping symbol may be inserted.

**E5** Situation Does the Timing Rule apply to a player writing an Equation?

Ruling Yes, it does. Each Equation-writer has two minutes to complete an Equation. After two minutes, a player who is not ready (and does not wish to concede) may take a one-point penalty and use an additional minute to complete an Equation. If an Equation is not complete after this additional minute, the player should concede since the Timing Rule says that he loses another point and forfeits what he is doing.

Penalty -1 for any Equation-writer taking an extra minute

Comment An Equation-writer who was ready when time expired but is waiting for a player who is taking an extra minute may not change his Equation during the extra minute. If he does so, then his Equation is automatically wrong since the change is illegal and the writer has in effect admitted the original Equation was incorrect. (See **E1** of this section.)

**E6** Situation Does the Timing Rule apply to player(s) checking an Equation?

Ruling Yes, it does. After two minutes, if a player is not ready to accept or reject an opponent's Equation, he may take a one-point penalty and use an additional minute. At the end of the additional minute, he should accept the Equation since, if this minute expires, he loses another point and forfeits what he is doing, which in this case means accepting the Equation.

Comment Suppose that, right before the end of the two minutes for checking, a checker says "I don't accept" but gives no reason. The checker must either take a -1 to obtain an extra minute to prove the Equation is incorrect or accept the Equation.

- E7** Situation In a three-player match, a player presents an Equation. May one opponent take two minutes to accept or reject the Equation and then the other opponent take an additional two minutes to accept or reject it?
- Ruling No; both opponents must check the Equation during the *same* two minutes. This means that, when more than one Equation must be checked, the players should agree to take one of them first and time two minutes for that Equation, with both opponents checking the third's Equation at the same time. Then two minutes should be allotted for checking another player's Equation and another two minutes if there is a third Equation.
- Comment The worst case would be all three players submitting Equations when no cubes remain in Resources. Checking could take *six* minutes.
- E8** Situation A player claims an opponent's Equation is wrong because the Equation-writer did not write the value of the Goal that the Equation equals.
- Ruling The writer does not have to write the *value* of the Goal. However, the writer must write his interpretation of the Goal. This includes placing grouping symbols to indicate the order of operations, indicating what any wild cube represents, whether a cube is used as an exponent (if the exponent variation is chosen), and where factorial signs are placed. In some instances, writing the interpretation of the Goal automatically gives its value, such as a Goal of 37 or the Goal 40 with 0 wild and the writer writes 45 to indicate what 0 represents. If the writer fails to do any of these in writing or if the Equation does not equal the written interpretation of the Goal, the Equation is incorrect. See Appendix A of the Tournament Rules for numerous examples of ways to indicate interpretations of Equations and Goals.
- E9** Situation A player claims that a symbol in an opponent's Equation (digit, operation sign, or grouping symbol) is not clear.
- Ruling If the symbol is ambiguous but could be interpreted as what the writer says it is, the official should rule in the writer's favor. However, if the symbol is clearly not what the writer claims it is, the official should rule in favor of the opponent. Other officials can be consulted for their opinion concerning the meaning of the disputed symbol(s).
- E10** Situation After a challenge, players X and Y present Equations. Z does not present an Equation. X's Equation is checked first and accepted by both opponents. Y's Equation is then checked. An error is found in Y's Equation and it is rejected. However, players realize that X's Equation contained the same or similar error. Is X's Equation rejected also?
- Ruling Yes, it is. There is no fixed order in which Equations are to be checked. X should not be correct simply because his Equation was checked first. Z scores 6 for the shake because he accepted X's Equation and is not presenting his own Equation.
- Comment This ruling applies only to a case where the first Equation has the same error as the second Equation. If X's Equation contained a different error from Y's, then the rejection of Y's Equation does not change the acceptance of X's. This ruling also implies an interesting strategy consideration for X. If he knows his Equation contains an error but it is accepted by opponents, he may not want to point out the same error in Y's Equation since his Equation will be retroactively ruled wrong.

- E11** Situation After an Impossible challenge, the Third Party does not present an Equation. The Challenger accepts the Mover's Equation. However, the Third Party finds an error in the Equation. Does the Challenger still receive 6 points even though he erroneously accepted the Equation?
- Ruling No, he does not. The Challenger scored 2 as soon as he accepted the Equation.
- Comment If the Third Party accepts the Equation and the Challenger subsequently finds an error, the Third Party scores 2 and the Challenger scores 6.
- E12** Situation Same as **E11** except that, after one opponent accepts the Equation, the other takes an extra minute under the Timing Rule before finding an error in the Equation.
- Ruling Same as for **E11**.
- Penalty -1 for the second opponent for taking an extra minute
- E13** Situation After accepting an opponent's Equation as correct, a player notices and points out an error in the Equation. The other opponent then rejects the Equation for this reason. Is this acceptable?
- Ruling Yes, it is. The Equation is incorrect and the shake is scored accordingly. The player who at first accepted the Equation still scores 2 if he did not present a correct Equation himself. The other opponent scores 4 or 6 (depending on the challenge and who is the Third Party).
- E14** Situation A player's Equation is rejected because it contains a cube that is not in Required, Permitted, or Resources. The Equation-writer claims the cube was turned over during the shake (or pushed into Forbidden from another section).
- Ruling In a two-player match, the official has no way of knowing whether the Equation-writer's claim is correct. In a three-player match, the official can rely on what two of the three players say. In either case, if the Equation-writer insists the cube has been turned over (or moved), the official could look at the papers of the opponent(s). If the opponent(s) also have the cube in question in their Equations, this would be strong evidence that the cube was available, and the official could rule in the Equation-writer's favor. In general, though, the official must rule on the basis of the present configuration of cubes on the mat unless there is strong evidence that a cube was turned over or shifted to another section. As a last resort, the official might order the shake replayed.
- E15** Situation A player writes an Equation like this:
- $$(5 + 1)(2) + 1 = 13$$
- Ruling The Equation is incorrect. It must be written as it would be made with the cubes, with a x sign between the pairs of parentheses and an additional set of grouping symbols like this:  $[(5 + 1) \times 2] + 1 = 13$
- E16** Situation A player writes an Equation like this:
- $$\frac{5 + 2}{2} + 1 = 9 \div 2$$
- Ruling The writer's attempt to use a fraction bar as a grouping symbol may be ignored by checkers. So a checker may rewrite the Solution to make it unequal to the Goal, like this:  $5 + (2 \div 2) + 1 = 7$ , which equals 7, not  $9 \div 2$ . Also the

fraction bar does not count as a division sign. So if a  $\div$  cube is in Required, the Equation is incorrect because it does not use all the Required cubes.

Comment The same ruling applies to a writer using a “built-up” fraction in his interpretation of the Goal or writing a fraction like this:  $\frac{3}{4}$

**E17** Situation A player writes a Equation like this:

$$\sqrt{3 + 1} - 2 = 8 - 8$$

Ruling The writer’s attempt to use the radical sign as a grouping symbol may be ignored by checkers. So a checker may rewrite the Solution to make it unequal to the Goal, like this:  $(\sqrt{3}) + 1 - 2 \neq 8 - 8$ .

Comment The same ruling applies to the illegal use of the radical sign as a grouping symbol in the writer’s interpretation of the Goal.

**E18** Situation When the two minutes for checking an Equation expires, a checker rejects the Equation but does not give a reason. While waiting for a judge, the same checker or the other one provides a valid reason for rejecting the Equation.

Ruling The Equation is incorrect. However, both checkers get a one point penalty for exceeding the time limit for checking the Equation.

Comment The ruling also applies to the situation where a checker states a different but valid reason for rejecting the Equation when the judge arrives after the two-minute checking time has expired.

**E19** Situation A checker calls a judge and says she does not accept the Equation because the Solution does not equal the Goal.

Ruling First determine if the checkers have made an effort to determine whether the Solution equals the Goal. If so, the judge will take the Equation and determine whether the Solution equals the Goal. Since ambiguity was not alleged by the checker(s), the judge will rule the Equation correct if one value of the Solution equals a legal value of the Goal.

**E20** Situation Same as **E19** except that the checker says the Solution does not *unambiguously* equal the Goal.

Ruling General claims of ambiguity are not allowed. Direct the checker(s) to copy the Equation to their paper and put symbols of grouping where they want. If either checker working independently or the two checkers working together can show a grouping that makes the Solution  $\neq$  Goal, the Equation is incorrect. Each checker has only *one* opportunity to show ambiguity.

Comment If the original time for checking the Equation has expired, each checker is penalized one point for taking an additional minute to show ambiguity.

**E21** Situation Same as **E20** except that, after no checker proves ambiguity, a checker still rejects the Equation because the Solution doesn’t equal the Goal.

Ruling If the extra minute for checking has expired, then no further objections may be made and the Equation is correct. However, if there is additional time available (with one or both of the checkers taking a -1 if necessary to obtain the extra minute), follow the Ruling of **E19**.

**E22** Situation When the time limit for checking player *C*’s Equation expires, checker *A* wishes to take additional time but checker *B* doesn’t.

Ruling By not taking an extra minute, *B* accepts the Equation as correct and scores 2 if he did not submit an Equation. *A* takes a one-point penalty in order to

check for an additional minute. If *A* still finds nothing wrong, the Equation is correct, and the shake is scored accordingly. If *A* shows the Equation is incorrect during the additional minute, *B*'s score doesn't change.

- E23** Situation The Equation-writer uses the expression  $\sqrt{4+5}$ . An opponent wants to interpret this as  $\sqrt{(4+5)}$ , which would make the Equation wrong.
- Ruling The opponent's interpretation is not allowed. In the absence of grouping symbols,  $\sqrt{\phantom{x}}$  applies to just the numeral immediately behind it. So  $\sqrt{4+5}$  must be interpreted as  $(\sqrt{4})+5$ , which equals 7.
- Comment The ruling applies to the expression whether it is in the Solution or the writer's interpretation of the Goal.
- E24** Situation The Goal is  $\sqrt{4+5}$  (ungrouped). May an Equation-writer interpret this as  $\sqrt{(4+5)}$  even though this is not the default interpretation for  $\sqrt{\phantom{x}}$ ?
- Ruling Yes. An Equation-writer may interpret an ungrouped Goal in any acceptable way. If the writer wishes  $(\sqrt{4})+5$ , writing  $\sqrt{4+5}$  in the Equation is sufficient since this is the default interpretation. However, the Equation-writer may also write  $\sqrt{(4+5)}$  to obtain the non-default meaning.
- E25** Situation A player challenges Now, and the Third Party decides to present an Equation. The Challenger presents an Equation. The Mover accepts the Equation before the Third Party points out an error in the Equation. The Third Party's Equation is also proved wrong by one of the opponents.
- Ruling All three players score 2 for the shake. By accepting the Challenger's (or Third Party's) Equation as correct, the Mover admitted, in effect, that he was wrong and therefore scores 2.
- Comment The Mover scored 2 *because he did not present an Equation* in this situation. If the Third Party had accepted the Challenger's Equation before the Mover found an error in it, the Third Party would not thereby have scored 2. The Third Party's score in this instance depended on the correctness of the Third Party's Equation.
- E26** Situation A player presents an Equation to the two opponents. One opponent claims the Solution does not unambiguously equal the Goal but presents an interpretation of the Solution and/or Goal that does not successfully prove ambiguity. The third player then presents a grouping of the Solution and/or Goal so that the Solution does not equal the Goal. Is this acceptable?
- Ruling If the third player proved ambiguity *during the same time* that the first checker unsuccessfully tried to prove ambiguity, then the Equation is Incorrect. However, if the third player discovered a grouping that made the Equation incorrect *after* the first checker presented his unsuccessful attempt, the Equation is correct, and no further objections to it are allowed.
- E27** Situation (**E** only) A player presents an Equation containing an expression like  $3\sqrt{5*6}$ . May an opponent group this as  $(3\sqrt{5})*6$ ?
- Ruling Yes, and this makes the Equation incorrect because  $3\sqrt{5}$  is not a whole number. The default interpretation of the expression is  $(3\sqrt{5})*6$  since, without parentheses,  $\sqrt{\phantom{x}}$  applies to just the number right behind it. The Equation-writer should have grouped the expression as  $3\sqrt{(5*6)}$ .
- Comment The ruling applies to the expression whether it is in the Solution or the writer's interpretation of the Goal.



## Section E: Writing and Checking Equations

### Part II: Situations Involving Variations

Note: The Equations Tournament Rules use the words Solution, Goal, and Equations in a particular way. The *Equation* has the form:

$$\text{Solution} = \text{Goal}$$

- E28** Variation Sideways  
Situation An Equation uses one 2 right-side up and another 2 sideways. Or a 2 is right-side up in the Goal and sideways in a Solution (or vice-versa).  
Ruling None of these uses of the 2 cubes is wrong. The sideways variation does not require a given numeral to be used consistently in the Equation.
- E29** Variation Sideways  
Situation An Equation has a sideways cube immediately behind or in front of another numeral.  
Ruling The Equation is incorrect. An operation sign must appear between the sideways cube and the other digit.  
Comment (**MJS** only) This assumes the Exponent variation is not in force. See **E50** below.
- E30** Variations Sideways, 0 Wild  
Situation May a Solution use a 0 as a 3 turned sideways to give 1/3?  
Ruling Yes, it may.  
Comment (**JS** only) The same ruling applies to x Wild.
- E31** Variation Upside-down  
Situation An Equation contains one 2 right-side up and another 2 upside-down. Or a 2 is right-side up in the Goal and upside-down in a Solution (or vice-versa).  
Ruling None of these uses of the 2 cubes is wrong. The upside-down variation does not require a given numeral to be used consistently in Goal and Solution.
- E32** Variation Upside-down  
Situation May a player, in a Solution, use 7<sup>5</sup> and interpret this as 7–5 (two)?  
Ruling No; upside-down 5 means *negative* 5. In 7–5, the – is a minus sign (subtraction) and hence does not conform to the meaning intended for the upside-down 5 by the upside-down variation. The Solution is incorrect.
- E33** Variation Upside-down  
Situation An Equation contains an upside-down cube immediately behind or in front of another numeral.  
Ruling The Solution is incorrect. An operation sign must appear between the upside-down cube and the other numeral.  
Comment (**MJS** only) This assumes the Exponent variation is not in force. See **E51** of this section.
- E34** Variation 0 Wild  
Situation An Equation uses one 0 for one number and another 0 for another number.  
Ruling The Equation is incorrect. The 0 Wild variation requires all 0's in the Equation represent the same number or (**MJS** only) the same operation.  
Comment The Equation-writer may indicate what 0 represents in *either* the Solution or the Goal. (**JS** only) This same ruling applies to x Wild.

- E35** Variation 0 Wild  
 Situation An Equation uses a 0 as a 5. An opponent argues that, since no 5's appeared in Resources, a 0 cannot stand for 5 for this shake.  
 Ruling The opponent's argument is erroneous. The variation allows 0 to stand for any numeral that is on the cubes whether that numeral appears in Resources or not.  
 Comment (**MJS** only) The same ruling applies to 0 used as an operation. (**JS** only) The same ruling applies to x Wild.
- E36** Variation 0 Wild  
 Situation An Equation-writer writes the Goal containing a 0 on his paper but does not specify what the 0 in the Goal equals.  
 Ruling The Equation-writer must specify in writing if a 0 in the Goal represents another numeral besides 0. However, if the writer specifies what a 0 in the Solution equals, the 0 in the Goal is presumed to have the same value (although the writer at times must indicate if a 0 in the Goal is upside-down or an exponent).  
 Comment See Appendix A of the Tournament Rules for a detailed discussion with numerous examples of when and how players must write their interpretation of the Goal.
- E37** Variation Number of Factors or (**E** only) Smallest Prime  
 Situation An Equation uses one x for Number of Factors (or Smallest Prime) and another x for multiplication. Does this make the Equation incorrect?  
 Ruling No; the Number of Factors (or Smallest Prime) variation does not rule out using x for multiplication. The position of the x in the Equation indicates its use. For example, in  $5xx(7-3)$  the first x means multiplication and the second means Number of Factors (or Smallest Prime).
- E38** Variation Number of Factors or (**E** only) Smallest Prime  
 Situation An Equation-writer uses the expression  $x7+9$ . An opponent wishes to interpret this as  $x(7+9)$ .  
 Ruling The opponent's interpretation is not allowed. With Number of Factors (or Smallest Prime), x applies just to the numeral immediately behind it unless grouping symbols are used. So  $x7+9$  means  $(x7)+9$ .
- E39** Variation Number of Factors, Factorial  
 Situation An Equation-writer uses the expression  $4+x7!$ . An opponent wants to interpret this as  $4+x(7!)$  which makes the Equation wrong.  
 Ruling The opponent may interpret  $4+x7!$  as  $4+x(7!)$ . The expression is ambiguous because it contains a conflict between the default interpretations for x as Number of Factors and !. By default, x as Number of Factors applies to just the numeral behind it. However, ! applies to just the numeral before it by default. So the Equation-writer must use parentheses to indicate the desired interpretation.  
 Comment (**E** only) The same ruling applies to x used for Smallest Prime.

- E40** Variation Multiple Operations  
 Situation After a challenge, may an Equation-writer use an operation cube in Resources multiple times?  
 Ruling Yes; after a challenge, all cubes in Resources are equivalent to cubes in Permitted. So, after a Now challenge, one operation cube (or a wild cube representing an operation) from Resources may be used multiple times. After an Impossible challenge, every operation cube (or wild cube representing an operation) in Resources may be used multiple times.
- E41** Variation Multiple Operations  
 Situation Must an Equation-writer indicate in writing that an operation sign is used more than once in the Solution?  
 Ruling No; if opponents ask, the Equation-writer may state orally which operation cubes are used multiple times although opponents should be able to figure this out for themselves.
- E42** Variation Factorial  
 Situation For a Goal of  $72 \times 10$ , a player presents this Equation:  $6! = 72 \times 10$   
 Ruling The Equation is incorrect because it does not contain at least two cubes. ! is not on a cube.
- E43** Variation Factorial  
 Situation An Equation-writer uses the expression  $4+7!$ . An opponent wants to interpret this as  $(4+7)!$   
 Ruling The opponent's interpretation is illegal. ! applies to just the numeral in front of it unless the Equation-writer uses grouping symbols to indicate otherwise. So  $4+7!$  means  $4+(7!)$ .
- E44** Variation Factorial  
 Situation An Equation-writer uses the expression  $\sqrt{9!}$ . An opponent wants to interpret this as  $\sqrt{(9!)}$ , which makes the Equation wrong.  
 Ruling The opponent may interpret  $\sqrt{9!}$  as  $\sqrt{(9!)}$ . The expression is ambiguous because it contains a conflict between the default interpretations for  $\sqrt{\quad}$  and !. By default,  $\sqrt{\quad}$  applies to just the numeral behind it. However, ! applies to just the numeral before it. So the Equation-writer must use parentheses to indicate the desired interpretation.
- E45** Variation (E only) LCM  
 Situation An Equation-writer uses one  $\sqrt{\quad}$  for LCM and another for root. Is this legal?  
 Ruling Yes, it is. The LCM variation does not rule out using  $\sqrt{\quad}$  for root.  
 Comment The same ruling would apply to the \* sign when GCF is chosen. Also each Equation-writer must specify in writing which  $\sqrt{\quad}$  stands for LCM (root is the default meaning) or which \* stands for GCF (exponentiation the default).
- E46** Variation (EM only) Percent  
 Situation An Equation-writer uses one  $\sqrt{\quad}$  upside-down for percent and another  $\sqrt{\quad}$  right-side up for root. Is this legal?  
 Ruling Yes, it is. The percent variation does not rule out using  $\sqrt{\quad}$  for root and does not require all  $\sqrt{\quad}$  signs to be used the same way.

- E47** Variation (EM only) Decimal Point  
 Situation An Equation-writer uses one \* as a Decimal Point and another for an exponent. Is this legal?  
 Ruling Yes, it is. The Decimal Point variation does not rule out using \* for exponentiation and does not require that all \* signs be used the same way. For this reason, players are encouraged to write a Decimal Point in the Equation when using an \* cube for a Decimal Point and \* when using it for exponentiation.
- E48** Variations (MJS only) Exponent  
 Situation An Equation-writer uses the expression  $4+3^2$ . An opponent wants to interpret this as  $(4+3)^2$ .  
 Ruling The opponent's interpretation is not allowed. The exponent applies to just the numeral in front of it unless the Equation-writer uses grouping symbols to indicate otherwise.  $4+3^2$  must mean  $4+(3^2)$ .
- E49** Variation (MJS only) Exponent  
 Situation An Equation-writer uses an \* followed by a digit of the exponent color.  
 Ruling There is nothing inherently wrong. The variation says that a cube *may* be used as an exponent without a \* cube.
- E50** Variations (MJS only) Exponent, Sideways  
 Situation May an Equation-writer use a sideways cube immediately behind or in front of another numeral?  
 Ruling A sideways cube could immediately precede or follow another numeral in two ways: if the sideways cube were the designated exponent color and used as an exponent; for example,  $4\curvearrowright$  means  $4^*(1/2)$ . Or if the sideways cube appeared immediately before an exponent; for example,  $\curvearrowleft 2$  means  $(1/4)^*2$ .
- E51** Variations (MJS only) Exponent, Upside-down  
 Situation May an Equation-writer use an upside-down cube immediately behind or in front of another numeral?  
 Ruling An upside-down cube could immediately precede or follow another numeral in two ways: if the upside-down cube were the designated color to be used as an exponent; for example,  $4\bar{\curvearrowright}$  means  $4^*(-2)$  with Red Exponent. Or if the upside-down cube appeared immediately before an exponent; for example,  $\bar{\curvearrowleft} 2$  means  $(-4)^*2$ .
- E52** Variations (MJS only) Exponent, Factorial  
 Situation An Equation-writer uses the expression  $\sqrt{x5!}^2$ . An opponent wants to interpret this as  $(x5!)^2$ , which makes the Equation wrong.  
 Ruling The opponent's interpretation is acceptable. The expression is ambiguous because it contains a conflict between the default interpretations for radical and for exponent and factorial. By default, the radical applies to just the numeral behind it. However, the factorial and exponent apply to just the numeral before them by default. So the Equation-writer must use parentheses to indicate the desired interpretation.

- E53** Variations **(MJS only)** Exponent, Number of Factors  
 Situation An Equation-writer uses the expression  $x12^2$ . An opponent wants to interpret this as  $(x12)^2$ , which makes the Equation wrong.  
 Ruling The opponent's interpretation is acceptable. The expression is ambiguous because it contains a conflict between the default interpretations for Number of Factors and exponent. By default,  $x$  as Number of Factors applies to just the numeral behind it. However, the exponent applies to just the numeral before it by default. So the Equation-writer must use parentheses to indicate the desired interpretation.
- E54** Variations **(MJS only)** Exponent, Number of Factors  
 Situation An Equation-writer uses the expression  $4+x12^2$ . An opponent wants to interpret this as  $(4+x12)^2$ , which makes the Equation wrong.  
 Ruling The opponent's interpretation is acceptable. The expression is ambiguous, as explained in the ruling for **E50** above. So an opponent may put parentheses in the expression as long as they do not violate a grouping the writer already indicated. Since the writer indicated no grouping, the opponent's interpretation is valid.  
 Comment The same ruling would *not* apply to the expression  $4+12^2$ . In this case, the 2 applies to just the numeral in front of it.  $4+12^2$  is not ambiguous and must mean  $4+(12^2)$ . See **E50** above.
- E55** Variation **(MJS only)** Powers of the Base  
 Situation An Equation-writer uses one 1 for one power of the base and another 1 for another power of the base. Is this legal?  
 Ruling Yes, it is. The variation does not require that all 1's represent the same power of the base.
- E56** Variation **(MJS only)** Multiple of  $k$   
 Situation An Equation equals the Goal.  
 Ruling The Equation is incorrect. The Multiple of  $k$  variation requires that a Solution differ from the Goal by a *non-zero* Multiple of  $k$ .
- E57** Variation **(MJS only)**  $k = 6$   
 Situation The Goal is a fraction; say,  $1 \div 2$ . May a Solution equal  $6 \frac{1}{2}$ ?  
 Ruling Yes, it may since  $6 \frac{1}{2}$  differs from  $\frac{1}{2}$  by 6, a Multiple of  $k$ . Other possible values for Solutions would be  $12 \frac{1}{2}$ ,  $-5 \frac{1}{2}$ , etc.  
 Comment If the Goal is a non-integral fraction, no Solution may equal an integer. The same applies to an irrational Goal like  $\sqrt{2}$ .
- E58** Variation **(MJS only)**  $k = 6$   
 Situation The Goal is 3. Is a Solution equal to  $-3$  correct?  
 Ruling Yes, it is.  $-3$  differs from 3 by  $-6$ , which is a multiple of 6.  $-9$ ,  $-15$ ,  $-21$ , etc., would also be correct values for Solutions.
- E59** Variation **(MJS only)** Base Eight  
 Situation An Equation-writer uses an "8" or "9."  
 Ruling The Equation is incorrect. In base  $m$ , the only digits are 0, 1, 2, 3, ...,  $m-1$ .

- E60** Variation (MJS only) Base Eight  
 Situation May an Equation-writer use 00, 01, 02, ..., 07?  
 Ruling Yes; these are two-digit numerals in Base Eight. (This situation would be especially interesting if 0 Wild were also in effect.)  
 Comment 00, 01, 02, ..., 09 are allowed in Goals whether base  $m$  is in effect or not.
- E61** Variation (JS only) Base Twelve  
 Situation Must an Equation-writer indicate in writing whether a  $\sqrt{\quad}$  in the Equation means root or eleven or a  $*$  means ten or exponentiation?  
 Ruling Yes, unless the position of the  $\sqrt{\quad}$  or  $*$  indicates its interpretation. Examples: In  $2\sqrt{3}$  and  $\sqrt{23}$ , each  $\sqrt{\quad}$  can mean only root since three-digit numerals are not allowed.  $2\sqrt{*}3$  may be interpreted as  $(2\sqrt{\quad})^*3$ , where the  $\sqrt{\quad}$  means eleven and  $*$  means exponentiation, or as  $2\sqrt{(\quad)^*}$  where  $\sqrt{\quad}$  means root and  $*$  means ten. So, in this case, the Equation writer must clearly indicate the desired interpretation in the Solution and/or the Goal.
- E62** Variations (S only) Imaginary  
 Situation The Goal is  $3|4$ . What is the default interpretation of this Goal?  
 Ruling  $|$  is ambiguous as to right-side up and upside down. The default is right-side up. So the default interpretation of  $3|4$  is  $12i$ .
- E63** Variation (S only) Imaginary  
 Situation The Goal is  $8|+3$ . May an Equation-writer interpret this Goal as  $8(|+3)$ ?  
 Ruling No; the Imaginary variation allows a numeral to be used immediately before or after  $|$  without a  $x$  sign. It does not allow a numeral to be used immediately before or after  $(|+3)$  without a  $x$  sign. So the Goal defaults to  $(8i) + 3$ .  
 Comment This ruling also means that no Solution may contain an expression like  $8(|+3)$ . Use  $8x(|+3)$  instead.
- E64** Variation (S only) Imaginary  
 Situation Is an expression like  $(0 - 0)|$  or  $|(8 + 5)$  allowed in the Equation?  
 Ruling Yes; the Imaginary variation allows  $|$  to be placed immediately before or after a numeral without a  $x$  sign. “ $0 - 0$ ” and “ $8 + 5$ ” are numerals since they name numbers. This includes expressions like  $||$  (which is  $\pm i^2$ ),  $|||$  ( $\pm i^3$ ), and so on, since  $i$  is itself a numeral.
- E65** Variation (S only) Imaginary  
 Situation An Equation-writer uses the expression  $\sqrt{4}$  or  $4^{(1\div 2)}$ . Are these expressions ambiguous?  
 Ruling  $\sqrt{4}$  is not ambiguous and equals the principal square root of 4, which is 2. However,  $4^{(1\div 2)}$  is ambiguous since, with Imaginary, all roots are possible with fractional exponents. So  $4^{(1\div 2)}$  can equal 2 or  $-2$ . The Equation-writer must indicate which value is desired. If not, an opponent may interpret the expression in a way that makes the Solution not equal the Goal.
- E66** Variation (S only) Imaginary  
 Situation Which of these expressions are ambiguous?  
 $3\sqrt{27}$ ,  $3\sqrt[3]{8}$ ,  $9^{\sqrt{9}}$ ,  $27^{\sqrt[3]{27}}$ ,  $\sqrt{9|}$ ,  $8^{(2\div 3)}$ ,  $|^{(3\div 4)}$   
 Ruling  $3\sqrt{27}$  defaults to 3.  $3\sqrt[3]{8}$  has no default.  
 $9^{\sqrt{9}}$  has no default.  $27^{\sqrt[3]{27}}$  has no default.  
 $\sqrt{9|}$  has no default whichever way it is interpreted:  $(\sqrt{9})|$  or  $\sqrt{(9|)}$   
 $8^{(2\div 3)}$  has no default.  $|^{(3\div 4)}$  has no default.

- E67** Variation (S only) Imaginary  
 Situation The Goal is  $3|*87$ . May this be interpreted as  $3x(|*87)$ ?  
 Ruling No, it may not. The variation allows a numeral right before or after  $|$  without an  $x$ ; it does not allow a numeral right before  $|*87$  without an  $x$ . This Goal must be interpreted as  $(3|)*87$ .
- E68** Variation (S only) Imaginary  
 Situation The Goal is  $8+3|$  or the Solution contains the expression  $8+3|$  without parentheses. May an opponent interpret  $8+3|$  as  $(8+3)|$ ?  
 Ruling No; the default order of operations for Imaginary matches the custom in math books. That is, the understood multiplication between the  $i$  and the numeral right before or after it takes precedence.  
 Comment See the Equations Tournament Rules for additional examples.
- E69** Variation (S only) Decimal in Goal  
 Situation The Goal is  $1 \div 8$ . An Equation-writer interprets the Goal as 12.5, arguing that  $1 \div 8 = .125$  and the decimal may be moved to any position.  
 Ruling The Equation is incorrect. This is not a valid interpretation of the Goal. The valid interpretations are:
- $$1 \div 8 (.125)$$
- $$.1 \div 8 (.0125)$$
- $$1 \div .8 (1.25)$$
- $$.1 \div .8 (.125)$$
- E70** Variation (S only) Decimal in Goal and Imaginary  
 Situation The Goal is  $4|3$ . May an Equation-writer place a decimal point in between the 4 and  $|$ ?  
 Ruling No, he may not. The part of the expression after the decimal point must be only one or more right-side up digits. A decimal may be placed in front of the 4 and/or in front of the 3.

## Section F: Combinations of Variations

- F1** Variations Sideways, Upside-down  
 Situation An Equation-writer uses a cube both sideways and upside-down in her Solution to give the negative reciprocal of the number.  
 Ruling This is acceptable as long as the Equation-writer indicates the use of sideways and upside-down properly.
- F2** Variations Sideways, 0 Wild  
 Situation An Equation-writer uses a 0 as, say, a 6 and indicates the 0 is sideways to represent  $1/6$ . Is this acceptable?  
 Ruling Yes, it is. Note that any other 0 in the Equation must be a 6 also. However, each 0 may be used right-side up or sideways independently of the other 0's.  
 Comment If Upside-down is also in force, one 0 in the Equation could be a right-side up 6, another 0 could be a sideways 6, and a third could be an upside-down 6. (**JS** only) The same ruling applies to x Wild.
- F3** Variations Upside-down, 0 Wild  
 Situation An Equation-writer uses a 0 as, say, a 6 and indicates the 0 is upside-down to represent  $-6$ . Is this acceptable?  
 Ruling Yes. Note that any other 0 in the Equation must be a 6 also but each 0 may be used right-side up or upside-down independently of the other 0's.  
 Comment If Sideways is also chosen, see the comment for **F2** above. (**JS** only) The same ruling applies to x Wild.
- F4** Variations Number of Factors or (**E** only) Smallest Prime, and Multiple Operations  
 Situation May an Equation-writer use an x cube multiple times as Number of Factors?  
 Ruling Yes; when used for Number of Factors (or Smallest Prime), x is still an operation sign and therefore may be used multiple times. In fact, the same x cube may be used one or more times for multiplication and one or more times for Number of Factors.
- F5** Variations Number of Factors or (**E** only) Smallest Prime, Multiple Operations  
 Situation In an expression like  $x^2$ , may the x be used multiple times in a Solution?  
 Ruling Yes, it may. For example, with Smallest Prime (**E** only),  $x^2$  may be interpreted as  $x^2$  (which is 3),  $xx^2$  (which is 5),  $xxx^2$  (which is 7), and so on. With Number of Factors,  $x^{24}$  may mean the number of factors of 24, which is 8, or  $xx^{24}$ , which is the number of factors of 8, which is 4, or  $xxx^{24}$ , which is the number of factors of 4, which is 3, or  $xxxx^{24}$ , which is 2 (and all further repetitions of the x give 2 from now on).  
 Comment An Equation-writer does not have to write out all the x's but may indicate how many times the x is to be repeated like this:  $x^2$   
 $\downarrow$   
10 x's
- F6** Variations Sideways, Number of Factors  
 Situation Is an expression like  $x^{\curvearrowright}$  legal in the Goal or Solution?  
 Ruling It is not legal because the number of factors operation may be applied to whole numbers only. A Goal like  $x^{\curvearrowright}x^6$  is legal because it can be grouped as  $x^{\curvearrowright}(x^6)$ .



- F7** Variations (**E** only) Sideways, Smallest Prime  
 Situation Is an expression like  $x^{\curvearrowright}$  legal in the Goal or Solution and, if so, how is it interpreted?  
 Ruling It is legal and means the Smallest Prime bigger than  $1/3$ , which is 2.
- F8** Variations (**E** only) LCM, Multiple Operations  
 Situation May an Equation-writer use a  $\surd$  cube multiple times as LCM?  
 Ruling Yes; when used as LCM,  $\surd$  is still an operation sign and therefore may be used multiple times. In fact, a  $\surd$  cube may be used one or more times for LCM and one or more times for root since the LCM variation does not require all  $\surd$  signs to mean the same operation.  
 Comment The same ruling applies to GCF and Multiple Operations.
- F9** Variations (**EM** only) Upside-down, Average  
 Situation May an Equation-writer use  $6-\overleftarrow{4}$  and interpret this as  $6+4$ ; that is, the average of 6 and 4 (five)?  
 Ruling No;  $6-\overleftarrow{4}$  is simply  $6-(-4)$  or ten.  $+$  must be explicitly used to obtain average.
- F10** Variations (**EM** only) Average, Multiple Operations  
 Situation May an Equation-writer use a  $+$  cube several times for average?  
 Ruling Yes; in fact, since the Average variation prohibits  $+$  from being used as addition, each time the  $+$  is used, it *must* mean average.
- F11** Variations (**M** only) 0 Wild, Average  
 Situation If a 0 cube is used as a  $+$ , must it stand for average?  
 Ruling Yes; the Average variation does not say the  $+$  *cube* stands for average.
- F12** Variations (**MJS** only) Sideways, Powers of the Base  
 Situation May a 1 cube be used sideways in the Goal or Solution to give the reciprocal of a power of the base?  
 Ruling Yes; however, the negative powers of the base are built into the Powers of the Base variation so that using the 1 sideways is redundant (but legal).
- F13** Variations (**MJS** only) Sideways, Base  $m$   
 Situation May an Equation-writer use a two-digit numeral in his Solution and claim both cubes are sideways to give something like  $1/12$ ,  $1/37$ , etc.?  
 Ruling No; with sideways, an operation sign must be used between the sideways numeral and another numeral.  
 Comment The only exception occurs when the Exponent variation is in force. The second of two consecutive sideways cubes (if it is the proper color) must be an exponent.
- F14** Variations (**MJS** only) Upside-down, Base  $m$   
 Situation May an Equation-writer use a two-digit numeral in his Solution and claim both cubes are upside-down to give something like  $-12$ ,  $-75$ , etc.?  
 Ruling No; with upside-down, an operation sign must be used between any upside-down numeral and another numeral.  
 Comment The only exception occurs when the Exponent variation is in force. The second of two consecutive upside-down cubes (if it is the proper color) must be an exponent.

- F15** Variations (**MJS** only) Upside-down, Powers of the Base  
 Situation May a 1 cube be used upside-down in the Goal or Solution and interpreted as the negative of a power of the base?  
 Ruling Yes; it may be interpreted as  $-1$ ,  $-10$ ,  $-100$ , ..., or  $-1$ ,  $-0.01$ ,  $-0.001$ , etc.  
 Comment The ruling applies if Base  $m$  is also chosen, with proper adjustment in the Powers of the Base.
- F16** Variations (**MJS** only) 0 Wild, Powers of the Base  
 Situation If a 0 is used as 1, may the 0 be interpreted as an integral power of the base?  
 Ruling Yes, it may. The Powers of the Base variation does not say the 1 *cube* may represent powers of the base.  
 Comment (**JS** only) The same ruling applies to x Wild.
- F17** Variations (**MJS** only) 0 Wild, Red Exponent  
 Situation Suppose the 0 on a red cube is used to represent a 9, which appears only on black cubes. May that red 0 still be used as an exponent without \* (or ^)?  
 Ruling Yes, it may. The Red Exponent variation says that “any red numeral may be used as an exponent ...” 0 is a red numeral regardless of which number it represents. However, a 0 on a blue cube may not be so used.  
 Comment (**JS** only) The same ruling applies to x Wild with the appropriate exponent color.
- F18** Variations (**MJS** only) 0 Wild, Base Eight  
 Situation May a 0 cube be interpreted as an 8 or 9?  
 Ruling No; the digits 8 and 9 do not exist in Base Eight. Also 0 may not represent “10” or “11” (the Base Eight equivalents of eight and nine).  
 Comment The same ruling applies to using a 0 as a 9 in Base Nine. (**JS** only) The same ruling applies to x Wild.
- F19** Variations (**MJS** only) 0 Wild, Base  $m$   
 Situation May a 0 cube represent a two-digit numeral?  
 Ruling No; the 0 may stand for any numeral that is “on the cubes;” there are no two-digit numerals on the cubes.  
 Comment (**JS** only) The same ruling applies to x Wild.
- F20** Variations (**MJS** only) 0 Wild, Multiple Operations  
 Situation May an Equation-writer use a 0 cube multiple times?  
 Ruling Only if the 0 stands for an operation. Also each 0 must represent the same operation symbol.  
 Comment (**JS** only) The same ruling applies to x Wild.
- F21** Variations (**MJS** only) Multiple of  $k$ , Base  $m$   
 Situation Is there any conflict between these variations?  
 Ruling No; for example, suppose the Goal is 24 (Base Eight), which is twenty. Solutions equal to twenty-six, thirty-two, ..., or fourteen, eight, two, negative four, etc., are acceptable. That is, first convert the Goal to base ten, then add or subtract multiples of  $k$ .

- F22** Variations (**MJS** only) 0 Wild, Number of Factors, Multiple Operations  
 Situation May an Equation-writer use the same wild cube for number of factors at one point and multiplication at another?  
 Ruling Yes, since the wild cube stands for  $x$  in each case.  
 Comment (**JS** only) The same ruling applies to  $x$  Wild.
- F23** Variations (**MJS** only) Powers of the Base, Base  $m$   
 Situation An Equation-writer writes an inconsistent explanation of the value of a 1. Example with Base Eight:
- $$\begin{array}{c} 10^8 \\ \downarrow \\ 1 \end{array}$$
- Ruling The Equation is incorrect. In the example, the base “10” is expressed in base eight while the exponent, 8, is in base ten since “8” is not a digit in base eight. The base and exponent must both be expressed in the same base. So in the example above, use either  $8^8$  or  $10^{10}$ .
- F24** Variations (**JS** only) 0 or  $x$  Wild, Base Eleven (or Twelve)  
 Situation May a wild cube be interpreted as an  $*$  (ten) or, in Base Twelve, as a  $\surd$  (eleven)?  
 Ruling Yes, it may since  $*$  is a “numeral on the cubes” in Base Eleven or Twelve and  $\surd$  is a numeral in Base Twelve.
- F25** Variations (**JS** only) 0 or  $x$  Wild, Base Eleven (or Twelve)  
 Situation The Goal is  $xx1+3$  or  $001+3$ . May both wild cubes be  $*$  signs to give  $**4 = 104 = 10000$ ?  
 Ruling Yes.
- F26** Variations (**JS** only) Multiple Operations, Base Eleven (or Twelve)  
 Situation May an Equation-writer use a  $*$  cube multiple times as the digit ten?  
 Ruling No; Multiple Operations allows an operation sign to be used multiple times. If used for the digit ten,  $*$  is not an operation sign. Another  $*$  could be used multiple times for raising to a power.  
 Comment The same ruling would apply to Multiple Operations and Base Twelve, with  $\surd$  used only once for the digit eleven.
- F27** Variations (**S** only) 0 or  $x$  Wild, Log  
 Situation May a wild cube be used as sideways  $\div$  to give the log interpretation?  
 Ruling Yes; the Log variation does not say the  $\div$  cube may mean log. All wild cubes must mean  $\div$ ; however, one wild cube may be used sideways for log and another wild cube may be division.
- F28** Variations (**S** only) Multiple of 6, Decimal in Goal  
 Situation The Goal is 62. If this Goal is interpreted as 6.2, would an Equation equal to 12.2 be correct?  
 Ruling Yes; other correct values for Solutions would be 18.2, 24.2, ..., and .2,  $-5.8$ ,  $-11.8$ , etc. Similarly, if the Goal were interpreted as .62, Solutions equal to 6.62, 12.62, etc., would be correct.
- F29** Variations (**S** only) 0 or  $x$  Wild, Imaginary  
 Situation May a wild cube be used for  $i$ ?

- Ruling Yes; with Imaginary,  $|$  is a *numeral* and a wild cube may represent any numeral on the cubes. Notice that, if one wild cube in the Equation represents  $i$ , then each wild cube must represent  $i$  or  $-i$  since  $|$  may be upside-down.
- F30** Variations (S only) 0 or x Wild, Imaginary  
 Situation May a wild cube be used as subtraction as well as  $i$  in the same Equation?  
 Ruling Yes because the wild cube stands for the same *symbol* everywhere it occurs whether it is right-side – or sideways –.
- F31** Variations (S only) Powers of the Base, Imaginary  
 Situation If the expression  $1|$  or  $|1$  appears in the Equation, may the 1 be interpreted as an integral power of the base?  
 Ruling Yes, it may. The expression may mean  $\pm i$ ,  $\pm 10i$ ,  $\pm 100i$ , ..., or  $(\pm 1/10)i$ ,  $(\pm 1/100)i$ , ...
- F32** Variations (S only) Multiple Operations, Imaginary  
 Situation May an Equation-writer use  $|$  multiple times?  
 Ruling No, since  $|$  represents the number  $i$ , not an operation.  
 Comment If a wild cube is used as  $-$ , it may be used multiple times. If used as  $|$ , only once.
- F33** Variation (S only) Imaginary  
 Situation May an Equation-writer use  $|$  upside-down to give  $-i$ ?  
 Ruling Yes; under the Imaginary variation,  $|$  becomes a “numeral cube” and therefore the upside-down variation allows it to be used upside-down.  
 Comment The same ruling applies to a wild cube used as  $|$ .
- F34** Variation (S only) Imaginary  
 Situation May the Goal or an Equation use  $|$  sideways for  $1/i$  (which equals  $-i$ )?  
 Ruling No, because sideways – means  $i$  or  $-i$ . However, there is no mathematical loss because  $1/i$  equals  $-i$ , which can be obtained by using  $|$  upside down.
- F35** Variation (S only) Imaginary  
 Situation In the Goal or Solution, may a numeral be placed right before or after  $|$  without a x sign?  
 Ruling Yes, it may. In fact, numerals may be placed both in front of and behind the same  $|$ , which can be interpreted as right-side up for  $i$  or upside-down for  $-i$ . The same principle also applies to a wild cube used as  $i$ .

**ON-SETS**  
**SITUATIONS**  
**2018-19**

## Guidelines for Officials for Judging the Correctness of On-Sets Solutions

The On-Sets Tournament Rules state that a Solution is correct unless an *opponent* shows that it violates one of the requirements for a correct Solution. Judges should therefore resist any attempt by a Solution-checker to get the judge to do the checker's job. Yet a decision must be made concerning the correctness of the Solution. The following guidelines are offered to help judges deal with situations where an opponent disputes a Solution but does not present a specific objection with which the judge can agree or disagree.

1. An official should never check whether a Solution uses the cubes correctly. However, the judge can answer specific questions about the cubes. Here are three examples.
  - a. Do all the cubes in Required have to be in the Restriction part? [Yes]
  - b. If B is wild and stands for R in one place in the Solution, does it have to stand for R in another place? [Yes]
  - c. After a Now challenge, may a Solution contain two cubes from Resources? [No]
2. If players cannot agree on the mathematical correctness of a Solution, the judge should tell the Solution-checker (or one of the checkers) to work out the Solution in the judge's presence. If there is a Restriction, the checker turns over any cards removed from the Universe, then selects the cards named by the Set-Name. The Solution-writer can object to the checker's manipulation of the cards at any point, and the third player (if there is one) may point out an error or help manipulate the cards. The judge oversees the process and settles specific controversies. The end result should be the correct number of cards in the Set-Name so that it is obvious whether the Solution equals the Goal. However, the process might be complicated by situations like the following ones.
  - a. The checker makes a mistake when working out the Solution. *The judge must not point out an error in the checker's work if no player notices the error.*
  - b. The Solution is ambiguous. The judge must not point this out. Since it is the checker who works out the Solution in the process suggested here, the checker can interpret the Solution so that it does *not* equal the Goal. If the checker interprets the Solution so that it equals the Goal, the judge must not point out the alternate (incorrect) interpretation. However, the third player may do so.
  - c. The players report that the checker has worked out the Solution and gets a different result from the writer. They want the judge to work out the Solution to settle the matter. In this case, the judge may do so.
  - d. The Solution-writer may volunteer to work out his own Solution. If this is acceptable to the checker(s), the judge may allow the writer to do so. If no opponent finds anything wrong with the way the writer works out the Solution and it equals the Goal, then opponents must accept the Solution. *The judge must not point out an error in the Solution-writer's work if no opponent notices the error.*
  - e. The Solution-checker may not be able to work out the Solution using the cards even with some prompting from the judge. Then the judge should ask the writer to demonstrate the Solution. If the writer is also confused, the judge may have to intervene.

## Section **OS-A**: Beginning a Shake And Selecting Variations

**NOTE A.** Many situations involving combinations of variations are covered by the comments following the variations in Section **XIII** of the On-Sets Tournament Rules.

**B.** Situations **A1-A19** in Section **A** of the Equations part of this manual also apply to On-Sets.

- A1** Situation During the first minute of the shake, the Goal-setter insists that the card dealer must deal the cards before the cubes are rolled (or vice-versa).  
Ruling This is incorrect. The two actions may occur in either order. Each player must complete his task within the one minute time limit.  
Penalty -1 for the Goal-setter if he does not roll the cubes within the one-minute time limit; -1 for the dealer if he does not deal the correct number of cards within the one-minute time limit.
- A2** Situation Depending on the division, the dealer puts out too few cards. The one-minute time limit expires (and a ten-second warning has been given).  
Ruling Illegal procedure; the dealer has an additional minute to add one or more cards to the Universe so that it contains a correct number of cards.  
Penalty The dealer loses one point; if the dealer does not complete the Universe correctly within an additional minute, the dealer loses another point and the dealing of the Universe moves to the player to the left of the dealer, which is the Goal-setter.  
Comment If the deal is transferred to the Goal-setter, all cards previously dealt are picked up, the Goal-setter reshuffles the cards, and deals a correct number of cards. Note that opponents are under no obligation to tell the dealer the correct number of cards that may be dealt. Nor should an official inform the dealer (unless all players at the table agree they want to know).
- A3** Situation The dealer puts out too many cards in the Universe.  
Ruling Illegal procedure; the dealer must remove last one or more cards that were dealt from the Universe so that it contains a correct number of cards. However, in general, cards dealt must remain in the Universe (just as a cube played to the mat must remain where it is played). Therefore, the dealer may remove only the minimum number of cards necessary. For example, in Elementary, Middle, or Junior Division, if the dealer dealt 14 cards, then only the last two may be removed to leave 12, the maximum number for these divisions.  
Penalty -1 for the dealer if he does not complete a correct Universe within the one-minute time limit.
- A4** Situation The dealer puts out too many or too few cards in the Universe. An opponent picks up the challenge block and challenges "Impossible."  
Ruling Illegal procedure; since no Goal has been set, no challenge can be made yet.  
Penalty -1 for the player attempting to challenge
- A5** Situation The dealer looks through the cards and deals selected ones.  
Ruling Illegal procedure; the dealer must shuffle the cards and deal from the top, just as a dealer of playing cards does. So this dealer must reshuffle and re-deal the cards.  
Penalty -1 for the dealer if a correct Universe is not redealt within the one-minute time limit

- A6** Situation The dealer does not shuffle all the cards before dealing. For example, the dealer does not put back into the deck the cards that were not dealt for the previous shake.  
Ruling The shuffle is incorrect and must be done over with all the cards included.  
Penalty -1 if the dealer's time expires before a correct deal is completed.
- A7** Situation The players pick variations before the cards are dealt.  
Ruling The dealer must now set out the Universe for the shake.  
Penalty -1 for the dealer for allowing variations to be selected with no Universe set.
- A8** Situation One player selects B Wild. Another chooses Y Required. However, no Y cube is in Resources.  
Ruling The second player's selection is illegal. Even if a wild cube has been chosen, the required cube variation demands that a Y *cube* be used in the Solution. A B cube standing for Y does not satisfy the Y Required variation.  
Penalty The player selecting the illegal variation loses one point and must pick another variation within 15 seconds.  
Comment The ruling is the same if both variations are chosen by the same player.
- A9** Situation A player selects Y Required when no Y is in Resources. No one charges illegal procedure and the next player makes a legal variation selection or sets the Goal. Then someone realizes that the Y Required selection was illegal.  
Ruling The first player's illegal procedure has been insulated. However, the Y Required variation should be ignored for the shake. Solutions do *not* have to contain a Y cube.
- A10** Situation Player A picks Y Wild or Y Required when no Y was rolled. Player B selects a valid variation. Then an opponent realizes that A's selection was incorrect.  
Ruling B's selection insulates A's illegal procedure. Therefore, A's selection stands but has no effect on the shake since there is no Y cube.  
Penalty None  
Comment If Y Required was called with no Y cube in Resources and the illegal variation is insulated, Y Required is set aside. So the Goal-setter should not call "no Goal," and no opponent should challenge "Impossible" if a Goal is set.
- A11** Situation (**E** only) The Goal-setter rolls the cubes without first setting out two  $\underline{V}$  cubes and one  $\underline{\Delta}$  (or vice-versa).  
Ruling The Goal-setter should turn over one or more Restriction cubes that were rolled so that two  $\underline{V}$  and one  $\underline{\Delta}$  (or one  $\underline{V}$  and two  $\underline{\Delta}$ ) are available.  
Penalty -1 if the Goal-setter does not adjust the cubes properly before the one-minute time limit expires
- A12** Situation (**E** only) The Goal-setter rolls the cubes without first setting out two  $\underline{V}$  cubes and one  $\underline{\Delta}$  (or vice-versa). No one notices the illegal procedure until variations are selected. Perhaps a Goal is set and one or more moves made before someone realizes the Goal-setter did not set the Resources properly.  
Ruling The Goal-setter's illegal procedure of not setting the Resources properly is insulated when a valid variation is selected by the player to the left of the Goal-setter. (The Goal-setter's variation selection cannot insulate his own illegal procedure.) However, = and  $\underline{C}$  cubes are not used in this division. So they should be treated like digit cubes and ignored or placed in Forbidden when someone realizes that the Goal-setter did not place them properly.



Comment If an opponent notices that the Goal-setter did not set the Restriction cubes properly before variations are selected (and the illegal procedure has not been insulated), the Goal-setter must turn over any = or C cube that was rolled to obtain two V and one Δ (or vice-versa).

**A13** Situation (**EM** only) One player chooses “B Wild.” Another then picks U and ∩ interchangeable even though neither cube is in Resources. Is the second player’s selection valid?

Ruling Yes, since a wild cube could be used for U or ∩. So the interchangeable variation can affect the shake.

Comment The same ruling applies if, after a wild cube is chosen, a player selects V and Δ interchangeable when neither cube is in Resources. The ruling also applies if the same player picks both variations. The order in which they are selected by the player makes no difference.

**A14** Situation (**EM** only) One player selects U Wild. Another player (or the same player when picking two variations) selects U and ∩ interchangeable. Are these variations contradictory?

Ruling No, they are not. If U is used just for U or ∩, it need not be used consistently. However, if U is used for any other symbol, it must be used only for that symbol throughout the Solution.

Comment The same ruling applies to V (or Δ) Wild and V-Δ interchangeable.

**A15** Situation (**MJS** only) A player selects no null Restrictions. However, no = or C cube is in Resources.

Ruling This is an illegal selection; the player must pick another variation. The only exception would be if a previous player (or the same player when selecting two variations) had picked a wild cube, which could then be used for = or C.

Penalty -1 for the player making the illegal variation selection

**A16** Situation (**JS** only) A player selects Required Card, but the chosen card is not in the Universe.

Ruling The player must pick another variation.

Penalty -1 for the player making the illegal variation selection

**A17** Situation (**JS** only) Same as **A16**. However, no one notices the illegal procedure, and the next player selects a legal variation and/or the Goal is set.

Ruling If a player selects a variation that cannot affect the shake and no one charges illegal procedure, that variation is ignored. There is no penalty against the player making the illegal selection since the selection was insulated by the next player’s action.

Comment If a Goal is set, and an opponent challenges Impossible, the challenge should be worked out in the usual way. The Goal-setter should write a Solution that ignores the Required Card variation. On the other hand, if the Goal-setter notices that the illegal selection has been insulated by the next player’s legal selection, the Goal-setter might call “no Goal.” However, an opponent should disagree with the declaration and set a Goal since the illegal variation is ignored.

- A18** Situation (JS only) A player selects Multiple Operations or one of the two interchangeable variations.  
 Ruling Since these variations are in effect at all times in Junior and Senior, this selection is illegal.  
 Penalty -1 for the player making the illegal variation selection  
 Comment The automatic variations are not listed on the variation selection sheet where players circle. Therefore, if they are using the proper selection sheet, this situation should not occur.
- A19** Situation (JS only) A player selects Blank Card Wild. However, the blank card is not in the Universe. The player selecting the variation claims that this selection means the blank card must now be added to the Universe.  
 Ruling The variation selection is illegal. Blank Card Wild may be chosen only if the blank card is in the Universe.  
 Penalty -1 for the player making the illegal variation selection
- A20** Situation (JS only) A player selects Blank Card Wild. Then another player selects "BR Required" even though that card is not in the Universe. The second player argues that the selection is legal and forces all players to put BR on the blank card.  
 Ruling The second variation selection is illegal, and that player must pick another variation. The Required Card variation requires that the card itself be in the solution set; the blank card may not be substituted for the required card.  
 Penalty -1 for the player making the illegal variation selection
- A21** Situation (JS only) A player selects Blank Card Wild. Then another player selects Blank Card Forbidden (or vice-versa).  
 Ruling The second variation selection is illegal, and that player must pick another variation.  
 Penalty -1 for the player making the illegal variation selection
- A22** Situation (JS only) A player chooses a Double Set that contains no cards.  
 Ruling The variation selection is illegal. The player must select another variation or select Double Set with a non-empty set that doesn't equal  $\underline{V}$ .  
 Penalty -1 for the player making the illegal variation selection
- A23** Situation (JS only) A player selects a Double Set that equals the Universe.  
 Ruling The variation selection is illegal. The player must select another variation or select Double Set with a different set.  
 Penalty -1 for the player making the illegal variation selection
- A24** Situation (JS only) A player selects  $R' \underline{U} (G - Y)$  as the Double Set.  
 Ruling The variation selection is illegal because the Set-Name contains more than four symbols (not counting the parentheses). The player must select another variation or select a different non-empty Double Set that doesn't equal  $\underline{V}$ .  
 Penalty -1 for the player making the illegal variation selection
- A25** Situation (JS only) A player selects Required (or Forbidden) Card. However, the chosen card is not in the Universe.  
 Ruling The variation selection is illegal. The player must select another variation or select Required (or Forbidden) Card with a card that is in the Universe.  
 Penalty -1 for the player making the illegal variation selection.

**A26** Situation (JS only) A player selects BR as the Required Card. Another player then selects BR as the Forbidden Card (or vice-versa).  
Ruling The second variation selection is illegal. The player must select another variation or select a different Forbidden Card.  
Penalty -1 for the player making the illegal variation selection.

## Section OS-B: Setting the Goal

*NOTE* Many situations in section **B-I** of the Equations manual also apply to On-Sets.

- B1** Situation The Goal-setter uses only one or two digit cubes in the Goal but does not move the remaining digit cube(s) to Forbidden.  
Ruling It is customary to place any unused digit cubes in Forbidden. However, the Goal-setter incurs no penalty for not doing so. If an opponent insists that the extra digit cube(s) be moved to Forbidden, the Goal-setter should do so.  
Comment If no challenge block is available, players may agree to use a digit cube not in the Goal as the challenge block.
- B2** Situation The Goal-setter sets an illegal Goal (e.g., an illegal configuration such as a backwards L). Instead of challenging Impossible, an opponent calls illegal procedure.  
Ruling The Goal is an illegal procedure. Since illegal procedure was charged before anyone challenged, the Goal-setter must revise the Goal to make it legal. All digit cubes played to the mat must stay. If fewer than three digits were used, one or more digits may be added to the Goal.  
Penalty -1 if the Goal-setter does not correct the Goal within the time limit
- B3** Situation Same as **B2** except that no opponent challenges or charges illegal procedure, and one or more cubes are played to Required, Permitted, or Forbidden.  
Ruling An Impossible challenge should be made against the latest Mover.  
Comment A judge should not tell the players that an Impossible challenge is in order unless the challenge has been made and the Third Party has taken a side.
- B4** Situation The Goal-setter sets a Goal that equals a negative number. An opponent charges illegal procedure.  
Ruling Assuming the Goal fits one of the acceptable configurations, it is not illegal procedure. Set aside the illegal procedure charge. The shake continues.  
Comment The official should not tell players that an Impossible challenge should be made against the Goal. However, if an Impossible challenge is made and players are arguing about the correctness of the challenge, the official may rule that no Solution is possible for a negative Goal. But do so only *after* the Third Party has taken a side.
- B5** Situation The Goal-setter sets the Goal before the dealer finishes dealing the cards.  
Ruling Setting the Goal is illegal procedure since it not only preceded the completion of the Universe but also the selection of variations. Remove the Goal from the mat until the Universe is complete and variations are chosen. The Goal-setter may then set the same or a different Goal.  
Comment Watch out for collusion between two players against the third player in this situation.

- B6** Situation (MJS only) The Goal-setter calls “Bonus,” plays an = or  $\underline{C}$  cube to Forbidden, then places one or more digit cubes on the Goal line. An opponent charges illegal procedure.
- Ruling The bonus move is illegal procedure and must be returned to Resources. The Goal-setter may not make another bonus move to replace the illegal one because he has started setting the Goal. Any cubes placed in the Goal must remain there.
- B7** Situation (E only) The Goal-setter sets out three  $\underline{V}$  or three  $\underline{\Delta}$  cubes before rolling the rest of the cubes. All players select variations and the Goal is set before a player notices there is no  $\underline{\Delta}$  (or  $\underline{V}$ ) cube in Resources.
- Ruling Failing to set two  $\underline{V}$  cubes and one  $\underline{\Delta}$  cube (or vice-versa) is illegal procedure by the Goal-setter. However, as soon as the player to the left of the Goal-setter chooses a valid variation, the illegal procedure is insulated. The shake proceeds with the Resources that were set and rolled.
- Penalty None
- Comment A player cannot insulate his own illegal procedure. Also, in this instance, dealing the cards cannot insulate failure to set the  $\underline{V}$  and  $\underline{\Delta}$  cubes properly since dealing and setting/rolling the Resources may happen in either order. If before or after the Goal-setter chooses a variation, an opponent notices the improper Resources, the Goal-setter must set the  $\underline{V}$  and  $\underline{\Delta}$  cubes properly.
- B8** Situation **The Goal-setter calls “No Goal.” What happens now?**
- Ruling **Opponents must either agree with the No Goal declaration or challenge it.**
- If all players agree, that shake is void, and the same player repeats as Goal-setter for a new shake.
  - An opponent who does not agree with the “No Goal” declaration indicates disagreement by picking up the challenge block and challenging the “No Goal” declaration. She then has two minutes to write a correct Solution (three minutes with Two Solutions in Senior Division). If there is a third player, he may also write a Solution. The Challenger and Third Party may use as many cubes from Resources as needed for the Solution. In this instance, the Challenger and Third Party must write the configuration of the digits for the Goal; for example, 3Z or an L-shape or upside-down T.
- Comment Scoring for this Challenge is as follows:
- If the Challenger presents a correct Solution, he scores 6. If the Challenger’s Equation is incorrect, he scores 2.
  - If the Third Party presents an incorrect Solution, she scores 2. If the Third Party presents a correct Solution, she scores 4 if the Challenger’s Solution is also correct or 6 if the Challenger’s Solution is incorrect. If the Third Party does not present a Solution, she scores 6 if the Challenger’s Solution is incorrect or 2 if the Challenger’s Solution is correct.
  - If either the Challenger or the Third Party presents a correct Solution, the original Goal-setter scores 2. If neither the Challenger nor the Third Party presents a correct Solution, the original Goal-setter scores 6.

## Section OS-C: Moving

*NOTE* Many situations in section C of the Equations manual also apply to On-Sets.

- C1** Situation A player moves an extra digit cube to Required, Permitted, or Forbidden.  
Ruling Since digits are not used in Solutions, this move does not count as the player's turn. The cube (and the remaining digit cube if it is still in Resources) should be placed in Forbidden. Any challenge against the digit cube move is set aside. It is still the turn of the same player.  
Penalty None against any player who challenged the move of the digit cube
- C2** Variation Required –  
Situation A player moves the last – cube in Resources to Forbidden. No – is in Required or Permitted. An opponent charges illegal procedure.  
Ruling The move is not illegal procedure. The cube stays in Forbidden where it was played.  
Comment An opponent should challenge Impossible. However, the official should not tell players this.
- C3** Variation Shift from Permitted  
Situation A player calls "Bonus," shifts a cube in Permitted to either Required or Forbidden, then makes a regular move. An opponent charges illegal procedure.  
Ruling The move is illegal procedure. A Bonus move is from Resources to Forbidden. The second cube must be returned to Resources and may not be replaced with another regular move on this turn.  
Penalty -1 if the Mover cannot undo the illegal procedure before his time expires.
- C4** Variation Shift from Permitted  
Situation A player makes a Bonus move to Forbidden, then shifts a cube in Permitted to either Required or Forbidden. An opponent charges illegal procedure.  
Ruling The move is not illegal procedure (assuming the player is not leading the match and the shift is not an = or C to Forbidden with more than four cubes in Resources).
- C5** Variation Shift from Permitted  
Situation A player moves the last cube in Resources to Permitted. May the next player shift the cube from Permitted to Required?  
Ruling No, he may not. Once the last cube is legitimately played to Required or Permitted, all moves are over, and all players must present a Solution (or challenge Impossible against the last Mover).
- C6** Variation Shift from Permitted  
Situation A player moves the last cube in Resources to Permitted. The player to the left of that player shifts the last cube from Permitted to either Required or Forbidden. An opponent challenges Impossible against the player who illegally shifted the last cube.  
Ruling Shifting the last cube out of Permitted is illegal procedure. The cube must remain where it was played. (See **C5** above.) The challenge is set aside, and players follow the Last Cube procedure.  
Penalty The player challenging the illegal shift loses a point.

- C7** Variation Shift from Permitted is *not* in force.  
 Situation A player moves a cube from Permitted to Required or Forbidden.  
 Ruling The move is illegal procedure. The cube is returned to Permitted and the player must make another move.  
 Penalty -1 if the player does not legally complete the turn before his time limit expires
- C8** Variation Shift from Permitted is *not* in force.  
 Situation A player moves a cube from Permitted to Required or Forbidden. An opponent challenges either Now or Impossible.  
 Ruling The move is illegal procedure. However, the illegal procedure is insulated by the challenge, which is worked out in the usual way. The cube shifted from Permitted stays where it was shifted.
- C9** Variations Shift from Permitted and Required –  
 Situation A – cube is in Permitted and there are no – cubes in Required or Resources. A mover shifts the – in Permitted to Forbidden. An opponent charges illegal procedure.  
 Ruling The move is not illegal procedure. The – cube stays in Forbidden.  
 Comment An opponent should challenge Impossible. However, the official must not tell players this.
- C10** Variation (**EM** only) Two operations (and not multiple operations or wild cube)  
 Situation Only one operation cube is in Required and Permitted. A player moves the last operation cube in Resources to Forbidden. An opponent charges illegal procedure.  
 Ruling The move is not illegal procedure. The cube stays in Forbidden where it was played.  
 Comment An opponent should challenge Impossible. However, the official must not tell players this.
- C11** Variation (**MJS** only) Required = (or C)  
 Situation With more than four cubes in Resources, a player moves the last = (or C) cube in Resources to Forbidden. No = (or C) is in Required or Permitted. An opponent charges illegal procedure.  
 Ruling The move is illegal procedure since = and C cubes may not be played to Forbidden until fewer than five cubes remain in Resources. The = (or C) in Forbidden is returned to Resources and the player must make another move.  
 Penalty -1 if the mover does not legally complete the turn before his time limit expires.  
 Comment An opponent should have challenged Impossible when the required cube was moved to Forbidden. However, the illegal procedure charge was made first, which allowed the mover to return the cube to Resources.
- C12** Variation (**MJS** only) Required = (or C)  
 Situation Same as **C11** except that four or fewer cubes are in Resources.  
 Ruling The move is not illegal procedure since = and C cubes may be played to Forbidden when fewer than five cubes remain in Resources.  
 Penalty None  
 Comment An opponent should challenge Impossible against the last Mover since no Solution that obeys the Required Cube variation is possible.

- C13** Variation (MJS only) Shift from Permitted  
Situation A player shifts an = or C cube from Permitted to Forbidden.  
Ruling The move is illegal procedure (even if fewer than five cubes are in Resources). The cube is returned to Permitted, and the player must make another move.  
Penalty -1 if the player does not legally complete the turn before his time limit expires  
Comment A player may shift an = or C cube from Permitted to Required.
- C14** Variation (MJS only) Shift from Permitted  
Situation Same as **C13** except that four or fewer cubes remain in Resources.  
Ruling The move is still illegal procedure. The cube is returned to Permitted and the player must make another move.  
Penalty -1 if the player does not legally complete the turn before his time limit expires



## Section **OS-E**: Writing and Checking Solutions

*NOTE:* Many situations in section **E** of the Equations manual also apply to On-Sets. Those situations are not repeated here.

- E1** Situation A player writes  $\emptyset$  in a Solution instead of  $\underline{\Delta}$ .  
 Ruling Since  $\emptyset$  is the standard symbol for the null set in mathematics books, the Solution should not be ruled incorrect just for this reason. However, the player should be told to use  $\underline{\Delta}$  for the empty set in the future.
- E2** Situation A player writes “=5” (or whatever the Goal is) after the Set-Name in the Solution.  
 Ruling The Solution is automatically wrong.
- E3** Situation A player checking the Set-Name  $R \underline{\cup} B$ ’ wants to insert parentheses to make it  $(R \underline{\cup} B)$ ’.  
 Ruling This placement of parentheses is illegal since the On-Sets order of operations requires ‘ to be done before  $\underline{\cup}$ .
- E4** Variations  $\underline{\cap}$  Wild,  $\underline{\cup}$ - $\underline{\cap}$  Interchangeable  
 Situation A player writes a Solution like this:  $B - R$  Is this acceptable?  

$$\begin{array}{c} \uparrow \\ \underline{\cup} \end{array}$$
  
 Ruling No, it is not.  $\underline{\cap}$  is wild, not  $\underline{\cup}$ . The Solution is incorrect.
- E5** Variations  $\underline{\cup}$ - $\underline{\cap}$  Interchangeable or  $\underline{\vee}$ - $\underline{\Delta}$  Interchangeable  
 Situation A Solution-writer writes  $\overline{\cap}$  or  $\overline{\Delta}$  to indicate a cube used upside-down.  
 Ruling Although not required to use the dash to indicate the bottom of the symbol, the Solution-writer has done nothing wrong. A problem would occur if there was no  $\underline{\cup}$  or  $\underline{\vee}$  cube available (that is,  $\underline{\cap}$  or  $\underline{\Delta}$  is actually being used upside-down). However, even in this case, the Solution should be accepted because, in general, the bars over or under the symbols are ignored in Solution.
- E6** **Situation A Solution contains an expression like this:  $B' (-G)$**   
**Ruling The Solution is automatically wrong because the expression makes no sense with the parentheses inserted as they are.**
- E7** Situation **(MJS only)** Only the Set-Name is circled on the Solution-writer’s paper. The Restriction is not inside the circle. (Or only the Restriction is circled and not the Set-Name.)  
 Ruling The Solution should not be ruled incorrect just for this reason. However, to avoid any further confusion, the Solution-writer should be told to circle both the Set-Name and the Restriction before checking begins.
- E8** Situation **(MJS only)** A player presents a Solution consisting of only a Restriction part.  
 Ruling The Solution is automatically incorrect.
- E9** Situation **(MJS only)** A player presents a Solution in which the Set-Name part is written above or to the left of the Restriction part.  
 Ruling There is nothing inherently wrong. There is no required order in which the parts must be written. Opponents will check the Restriction part first regardless of the order in which the two parts of the Solution are listed.

- E10** Situation (MJS only) A player presents a Solution like B = B, B. An opponent claims the Solution does not contain at least two cubes.  
Ruling Since the Restriction is part of the Solution, this Solution contains three cubes. The opponent's objection is rejected.
- E11** Variation (MJS only) B required  
Situation A Solution-writer uses a B cube only in the Restriction part.  
Ruling If there is a B cube in Required, the Solution is incorrect since any color or operation cube in Required must be used in the Set-Name. However, if the B cube is in Permitted or Resources, the Solution-writer's use of the B cube satisfies the variation since it states only that the *Solution* must contain a B cube. A Solution consists of the Set-Name and, if one is made, a Restriction.
- E12** Variation (MJS only) = (or C) Required  
Situation A Solution-writer presents a Solution consisting only of a Set-Name.  
Ruling The Solution is incorrect. Once = (or C) required is chosen, all Solutions must include a Restriction part whether = (or C) is played to Required or not.
- E13** Variation (MJS only) No null Restrictions  
Situation A Solution-writer presents a Solution containing two Restrictions and tries to specify which one must be worked out first.  
Ruling Each Restriction must remove at least one card regardless of which one is applied to the Universe first. The writer may not specify an order.
- E14** Variation (MJS only) No null Restrictions  
Situation No = or C cube is in Required. A player presents a Solution consisting of only a Set-Name. An opponent argues that the Solution does not obey the no null Restrictions variation.  
Ruling The opponent is incorrect. No null Restrictions does not require that a Restriction be made. It says that *if* any Restriction is made, it must remove at least one card from the Universe.
- E15** Variation (MJS only) Y Wild  
Situation A Solution-writer indicates that Y stands for G in the Restriction. However, he does not show explicitly that Y also stands for G in the Set-Name.  
Ruling The Solution is not automatically incorrect. Since the wild cube must stand for the same symbol everywhere in a Solution, the presumption is that Y stands for G throughout the Solution. So the Solution-writer may indicate the use of the Y in writing in *either* the Set-Name or the Restriction.
- E16** Variation (MJS only) Two operations  
Situation A Solution-writer uses two operations in a Restriction but not in the Set-Name or one operation in the Restriction and another in the Set-Name.  
Ruling The Solution is incorrect. The variation requires two operations in the Set-Name.
- E17** Variation (JS only) Blank Card Wild  
Situation A Solution-writer does not specify which color(s) are on the blank card.  
Ruling The Solution is not automatically incorrect. However, in checking the Solution, opponents should assume that *no* colors are on the blank card. Any attempt by the writer to state orally which colors are on the blank card should be ignored.

- E18** Variation (JS only) Blank Card Wild  
 Situation A Solution is written near the bottom of the paper. At or near the top, the player copied the Universe and on the blank card of that Universe indicated which colors are on the blank card. An opponent objects, saying that the colors on the blank card must be indicated with or near the Solution.  
 Ruling The Solution should not be ruled incorrect simply for this reason. The variation says that the Solution-writer must indicate in writing which colors are on the blank card. The writer has done that.  
 Comment If necessary, the judge may ask the writer to specify which blank card designation on his paper counts or whether the writer wants the blank card to be blank.
- E19** Variations (JS only) Blank card required and Blank Card Wild  
 Situation A player presents a Solution in which the Set-Name contains the blank card with one or more color dots on it.  
 Ruling This does not in itself make the Solution incorrect. Blank card required means that the blank card must be in the Set-Name. Then Blank Card Wild allows the Solution-writer to put one or more colors on it.
- E20** Variations (S only) Two Solutions  
 Situation After a Now challenge, a player presents two Solutions. One uses, say, a B cube from Resources and the other uses a R cube from Resources.  
 Ruling There is nothing inherently wrong with this. The Solutions are checked separately. As long as each one uses no more than one cube from Resources, they obey the rules for a Now challenge.
- E21** Variations (S only) Two Solutions  
 Situation After an Impossible challenge, a player presents two Solutions. One uses a certain combination of cubes from Resources while the other uses a different combination of Resource cubes.  
 Ruling There is nothing inherently wrong with this. The Solutions are checked as if they were presented by different players. There is no requirement that both Solutions use the same Resource cubes (or same Permitted cubes).
- E22** Variations (S only) Two Solutions  
 Situation The Goal is 0. A player challenges Impossible.  
 Ruling The Challenger is correct. There is no way to present two Solutions for a Goal of 0 and have one solution set contain at least one card that is not in the other solution set.  
 Comment The official should not state this ruling until the Third Party has taken a side.
- E23** Variations (S only) Two Solutions  
 Situation The Goal equals the number of cards in the Universe. A player challenges Impossible.  
 Ruling The Challenger is correct. There is no way to present two Solutions for this Goal and have one solution set contain at least one card that is not in the other solution set.  
 Comment The official should not state this ruling until the Third Party has taken a side.

- E24** Variations (**S** only) Two Solutions, double set  
 Situation The Goal is larger than the number of cards in the Universe. However, to equal the Goal, all cards in the Universe must be in the solution set. A player challenges Impossible.  
 Ruling Same as **E23**  
 Comment Same as **E23**
- E25** Variations (**S** only) Two Solutions, Wild cube  
 Situation A player presents a pair of Solutions. In one Solution, the wild cube stands for U. In the other Solution, the wild cube stands for –.  
 Ruling There is nothing inherently wrong with this. The Solutions are judged separately. As long as *within* each Solution, the wild cube stands for the same symbol everywhere, the Solution-writer has satisfied the wild cube variation.
- E26** Variations (**S** only) Two Solutions, Blank Card Wild  
 Situation A player presents a pair of Solutions. The two solution sets contain the same cards except that for one Solution the blank card contains B and R dots and for the other the blank card contains G and Y dots (or no dots).  
 Ruling The Solution-writer is incorrect. Both Solutions yield the same set of physical cards from the Universe and therefore do not satisfy the requirement of the two Solutions variation.
- E27** Variations (**S** only) Two Solutions, Blank Card Wild  
 Situation The Goal is 1. A player presents two Solutions. The solution set in each case consists of the blank card but with different sets of dots.  
 Ruling Same as **E26**
- E28** Variations (**S** only) Two Solutions, Required Card  
 Situation The Goal is set as 1. A player challenges Impossible.  
 Ruling The Challenger is correct. There is no way to present two Solutions for this Goal and satisfy both the Two Solutions and Required Card variations.  
 Comment The official should not state this ruling until the Third Party has taken a side.
- E29** Variations (**S** only) Two Solutions, Required Card, Double Set  
 Situation The Required Card is part of the Double Set. The Goal is set as 2. A player challenges Impossible.  
 Ruling The Challenger is correct. There is no way to present two Solutions for this Goal and satisfy all the variations.  
 Comment The official should not state this ruling until the Third Party has taken a side.
- E30** Variations (**S** only) Two Solutions, Absolute Value  
 Situation The Goal contains at least one upside-down cube. A player presents two Solutions. One of them equals the Goal with the upside-down cube representing a negative number while the other Solution equals the Goal using the absolute value of the upside-down digit.  
 Ruling There is nothing inherently wrong with this. Since the Solutions are separate, each one may equal a different value of the Goal.