2019-20

On-Sets

Tournament Rules

On-Sets Tournament Rules 2019-20

TABLE OF CONTENTS

NOTES:

1. All quotations in these Tournament Rules are from the rules in the booklet that accompanies the game kit.
2. The rules are formatted so that someone reading them for the first time can concentrate on the section in the largest font (same size as this text). The smaller text gives explanations and comments that explicate the previous rule or procedure but can be skipped on a first reading.
3. Changes from last year's rules are highlighted in **bold**.
4. Blue words and phrases are defined in the On-Sets Glossary, which is included at the end of this document.

BASIC ON-SETS®

Basic On-Sets can be introduced with the following seven rules. The complete Official Tournament Rules (including Restrictions in all divisions except Elementary) should be used once players become familiar with the game.

I. GOAL Rule

Two- or three-player matches will be played. To start, the player rolling the cubes first puts out two V and one ∧ cube or two ∧ and one V, then rolls the remaining 15 cubes. The symbols that show on the top faces of the cubes form the *Resources*. While this player is rolling the cubes, the player to his right shuffles and deals the cards (10 to 14 in Senior Division, 6 to 12 in all other Divisions). The cards dealt form the *Universe*.

A. The player who rolled the cubes sets a Goal by moving one or more digit cubes from the Resources to the Goal section of the playing mat.

B. If more than one cube is used to set the Goal, the way the cubes are placed determines the Goal's value.

 1. The sum of two numbers is indicated by placing the cubes side by side.

 2. The product of two numbers is indicated by placing the cubes in a vertical line.

 3. The negative of a number is indicated by placing the numeral upside-down.

II. MOVE Rule

After the Goal has been set, play progresses to the left. When it is your turn to play, you must either challenge (see rule **IV**) or move a cube from Resources to the Forbidden, Permitted, or Required section.

III. SOLUTION Rule

A Solution, which is written on paper, is the name of a set. The number of Universe cards in the set must equal the Goal. The Solution must obey these requirements.

A. The Solution must contain at least two cubes.

B. It must use *all* cubes in Required.

C. It must use *no* cubes in Forbidden.

D. It may use none, some, or all of the cubes in Permitted.

E. In the absence of grouping symbols, the ' operation takes priority over the binary operations (U, ∩, and –). If necessary, the Solution-writer must use parentheses or other grouping symbols to indicate the order of operations. The Solution is incorrect if it has an interpretation that does not equal the Goal.

IV. CHALLENGE Rule

A. Whether or not it is your turn, you may challenge another player who has just set the Goal or moved. To do so, you must pick up the challenge block and say one of the following.

 1. NOW: This means that the Challenger claims a Solution can be written using:

  a. all the cubes in Required and

  b. none, some, or all of the cubes in Permitted and

  c. *one* more cube from Resources, if needed.

 2. IMPOSSIBLE: This means the Challenger claims that nobody can write a Solution that satisfies all the requirements placed on the cubes in the Forbidden and Required sections, no matter how many cubes might still be used from the Permitted section and the Resources.

B. After a challenge in a three-player match, the Third Party (the player who is neither the Challenger nor the last Mover) *may* present a Solution but does not have to.

V. CORRECTNESS Rule

After a challenge, a player is correct if and only if that player

A. has to write a Solution or may write a Solution and does so correctly or

B. does not have to write a Solution (someone else does) and nobody writes a correct Solution.

VI. CHALLENGE-SCORING Rule

A. Any player who is not correct scores 2.

B. A correct Challenger or Mover scores 6.

C. A correct Third Party scores 6 if she **(a)** presents a correct Solution after an Impossible challenge or **(b)** presents a correct Solution after a Now challenge and the Challenger does *not* present a correct Solution, or **(c)** does not present a Solution after an Impossible challenge, and the Challenger's Solution is incorrect. Otherwise, a correct Third Party scores 4.

VII. LAST CUBE Rule

As soon as there is only one cube left in Resources, the only challenge allowed is IMPOSSIBLE. If nobody challenges IMPOSSIBLE, the player whose turn it is must move the last cube to either Required or Permitted. Then, unless someone makes an IMPOSSIBLE challenge, all players write a Solution. Whoever is correct scores 4 points; whoever is incorrect, scores 2.

ON-SETS® Tournament Rules 2019-20

I. Starting a Match (Round)

    A. Two- or three-player matches will be played. A *match* is composed of one or more shakes. A shake begins with the rolling of the cubes, the dealing of some cards, and the setting of a whole number as the Goal for that shake. A shake ends with at least one player attempting to write a *Solution* which both equals the Goal and correctly uses the cubes on the playing mat.

    B. The following equipment is needed to play the game.

        1. 16 cards: each card contains a unique combination of zero to four dots colored blue (B), red (R), green (G), or yellow (Y). No card contains more than one dot of any color. At the start of a shake, some of these cards are dealt face up to form the Universe for that shake.

        Comment   Players should make sure all 16 cards are in the game, with no duplicates. One of the cards is blank.

        2. 18 cubes: these consist of the following groups.

            a. 3 digit cubes: each face has one of the digits 1 through 5.

            Comment   The digit cubes are used only in setting the Goal.

            b. 8 color cubes: each face has a dot colored B, R, G, or Y. Each dot names the set of all cards in the Universe which contain a dot of that same color.

            c. 4 operation cubes: each face has one of the symbols U̲, ∩̲, –, or '.

                (i)   U̲ means the union of two sets.

                Example   B U̲ G is the set of cards in the Universe which are *either* B or G.

                (ii) ∩̲ means the intersection of two sets.

                Example   R ∩̲ Y is the set of cards which are *both* R and Y.

                (iii) – means set subtraction.

                Example   B – Y is the set of cards which are B *but not* Y.

                (iv) ' means the *complement* of a set.

                Example   G' (often read "green prime") is the set of cards which are *not* G.

            d. 3 Restriction cubes: each face has one of the symbols V̲, ∧̲, =, or C̲.

                (i)   V̲ names the set of all the cards in the Universe for the shake.

                (ii) ∧̲ names the set of no cards (the null or empty set).

                (iii) = and C̲ are special operators used to make mathematical statements about the cards in the Universe. Each such statement is called a Restriction. (See section **VI-B** below.)

                Comment   The = and C̲ symbols are not used in Elementary Division.

        3. A playing mat: this contains four sections.

            a. Goal: digit cubes played here form the Goal.

            b. Required: all cubes played here *must* be used in any Solution.

            c. Permitted: any or all cubes played here *may* be used in any Solution.

            d. Forbidden: *no* cube played here may be used in any Solution.

            Comment   Many games have a section labeled "Resources." However, any reference in these rules to the "playing mat" or the "mat" does not include the Resources section.

        4. A one-minute sand timer: this is used to enforce time limits.

5. A challenge block: This is a cube or similar object and not a flat object like a coin. It should not be so large that two players can grab it simultaneously.

C. Players may use only pencils or pens, blank paper, and (for Adventurous On-Sets) variation sheets. No prepared notes, books, tables, calculators, cell phones or other electronic devices may be used except that players' paper may contain approved preprinted Universe charts on which the cards that are dealt may be marked.

> *Comment* The chart a player uses may not have sets pre-shaded or pre-marked in any way. (See Appendix **B** for samples.)

D. The Goal-setter for the first shake is determined by lot. On each subsequent shake, the Goal-setter is the player immediately to the *left* of the previous Goal-setter.

> To determine the first Goal-setter, each player rolls a digit cube. The player rolling the highest digit sets the first Goal. Players tied for high digit roll again until the tie is broken.

II. Starting a Shake

A. To begin a shake, the Goal-setter rolls all 18 cubes. The symbols on the top faces of the rolled cubes form the Resources for the shake.

1. A shake begins as soon as the timing for rolling the cubes and dealing the cards is started or the cubes are rolled or the first card is dealt.

2. During a shake, no player may turn over a cube or obstruct the other players' view of any cube. (See section **IX-C**.)

3. In Elementary Division, the three Restriction cubes are not rolled. Instead the Goal-setter first sets out either two $\underline{V}$ and one $\underline{\wedge}$ cube or one $\underline{V}$ and two $\underline{\wedge}$ cubes. Then the remaining cubes are rolled.

B. During the Goal-setter's time to roll the cubes, the player to the *right* of the Goal-setter shuffles and deals the cards.

1. In Elementary, Middle, and Junior Divisions, at least six but no more than 12 cards must be dealt.

2. In Senior Division, at least 10 but no more than 14 cards must be dealt.

> In all divisions, after the cards are dealt and positioned in a manner agreeable to all players, no one may touch them or in any way obstruct the other players' view of them until Solutions are checked. (See section **VII-C**.) However, players may look at the cards that were not dealt (usually for purposes of marking their charts).

> *Comment* The dealer may not take back a card that has been dealt unless the number of cards exceeds the maximum allowed in the division. In that case, the extra card(s) – the last one(s) dealt - must be removed from the Universe.

C. In Adventurous On-Sets, after the cubes have been rolled and the cards have been dealt but before the Goal is set, each player must select a variation from the appropriate list in section **XIII** of these rules. A variation is a special rule which, if it conflicts with any of the regular tournament rules, supersedes those rules.

1. The Goal-setter makes the first selection, then the player to the left of the Goal-setter, then the third player if there is one.

a. Each player has 15 seconds to make a variation selection.

b. To begin a shake, the Goal-setter has one minute to roll the cubes. At the end of this minute, she has 15 seconds to select a variation. However, if the Goal-setter selects a variation before the minute for rolling the cubes expires, the next player has the rest of that minute plus 15 seconds to select a variation. If the second player also selects a variation before that minute expires, the third player (if there is one) has the rest of that minute plus 15 seconds to select.

c. A player selects a variation by circling its name in the list for that shake. This list is located on the reverse side of the scoresheet or on a separate sheet. For certain variations (e.g., Required Cube, Wild Cube, Double Set), the player must also fill in a blank to indicate which cube is required or wild, which set counts double, and so on.

2. If a player selects a variation that has no effect on the shake, a variation that conflicts with one already chosen for the shake, or a variation that has already been chosen for the shake, the player loses one point and must pick another variation. If, on the second try, the player still does not select an appropriate variation, he loses another point and may not pick a variation for that shake.

If a player's illegal variation selection is not pointed out before the next player selects a legal variation or a legal Goal is set (whichever comes first), the player making the illegal selection is not penalized. However, the illegal variation is ignored for the shake.

Examples   It is illegal to choose U Required when no U cube was rolled, Y Wild when no Y cube is in Resources, or (in Middle, Junior, and Senior) No Null Restrictions when no = or C cube was rolled (and no Wild Cube has been chosen).

3. In two-player matches in Elementary, Middle, and Junior Divisions, the player who is not the Goal-setter must select *two* variations for the shake. In Senior Division, any player may pick two variations for any shake in both two- and three-player matches.

A player picking two variations must select both within the 15 second time limit. (See Section **XI-A-1-b**.)

III. Setting the Goal

A. The player who rolls the cubes must set a Goal by transferring the cube(s) of the Goal from Resources to the Goal section of the playing mat.

B. A Goal consists of at least one and at most three digit cubes that form an expression that names a whole number.

1. If more than one cube is used to set the Goal, the way the cubes are placed in the Goal determines the Goal's value.

a. The sum of two numbers is indicated by placing the cubes in a horizontal line (side by side).
b. The product of two numbers is indicated by placing the cubes in a vertical line.
c. The negative of a number is indicated by placing the cube so that its numeral is upside-down.

The following are the only legal configurations for the Goal.

Goal	Meaning	Goal	Meaning
A	A	A B C (vertical)	A x B x C
A B	A + B		
A B C	A + B + C	A / B C	A x (B + C) or (A x B) + (A x C)
A / B (vertical)	A x B	A / B C	(A x B) + C

Comment Any digit cubes not used in the Goal should be placed in Forbidden since they are not used in Solutions.

2. Once a digit cube touches the Goal section of the mat, it must be used in the Goal.

a. The Goal-setter indicates the Goal has been set by saying "Goal."

b. The Goal-setter may rearrange or regroup the cubes in the Goal section until she says "Goal."

c. The Goal may not be changed once it has been set.

C. Before moving the first digit cube to the Goal section of the mat, the Goal-setter may make a bonus move.

1. To make a bonus move, the Goal-setter must say "Bonus," then move one non-digit cube (but not = or <u>C</u>) from Resources to Forbidden.

2. A Goal-setter who is leading in the match may not make a bonus move.

If the Goal-setter makes a bonus move while leading in the match and an opponent points out the error before the next player moves or someone legally challenges, the cube in Forbidden is returned to Resources. The Goal-setter is also penalized one point.

D. If the Goal-setter believes no Goal can be set which has at least one correct Solution (see section **VII**), he may declare "no Goal." Opponents have one minute to agree or disagree with this declaration.

1. If all players agree, that shake is void, and the same player repeats as Goal-setter for a new shake.

Comments

(a) The Goal-setter would declare "no Goal" only in those rare instances when an unusual set of Resources was rolled. For example, there are three 1's, the operations are all <u>U</u> signs, and each color appears on at least four cards. (Even in this case, the Goal-setter might be able to pick a variation like wild cube that would allow a Goal to be set.)

(b) Players receive no points for the void shake.

(c) If the Goal-setter makes a Bonus move, he is committed to setting a Goal and may not declare "no Goal."

2. An opponent who does not agree with the "No Goal" declaration indicates disagreement by picking up the challenge block and challenging the "No Goal" declaration. She then has two minutes to write a correct Solution (three minutes with Two Solutions in Senior Division). If there is a third player, he may also write a Solution. The Challenger and Third Party may use as many cubes from Resources as needed for the Solution. In this instance, the Challenger and Third Party must write the configuration of the digits for the Goal; for example, 3⌐ or an L-shape or upside-down T. Scoring for this Challenge is as follows:

- If the Challenger presents a correct Solution, he scores 6. If the Challenger's Equation is incorrect, he scores 2.
- If the Third Party presents an incorrect Solution, she scores 2. If the Third Party presents a correct Solution, she scores 4 if the Challenger's Solution is also correct or 6 if the Challenger's Solution is incorrect. If the Third Party does not present a Solution, she scores 6 if the Challenger's Solution is incorrect or 2 if the Challenger's Solution is correct.
- If either the Challenger or the Third Party presents a correct Solution, the original Goal-setter scores 2. If neither the Challenger nor the Third Party presents a correct Solution, the original Goal-setter scores 6.

IV. Moving Cubes

A. "After the Goal has been set, play progresses in a clockwise direction" (to the left).

B. When it is your turn to play, you must either move a cube from Resources to one of the three sections of the playing mat (Required, Permitted, Forbidden) or challenge the last Mover.

The move of a cube is completed when it touches the mat. Once a cube is legally moved to the mat, it may not be moved again during the shake. (Exception: When the Shift from Permitted variation is played – see section **XIII** below.)

C. If you are not leading in the match, then "on your turn you may take a bonus move before making a regular move."

    1. To make a bonus move, the Mover must say "Bonus," then move a cube from Resources to Forbidden.

        Comments

        (a) "If you do not say 'Bonus' before moving the cube to Forbidden, the move does not count as a bonus move but as a regular move to Forbidden." You are not entitled to play a second cube.

        (b) When making a bonus move, the first cube *must* go to Forbidden. The second cube may be moved to Required, Permitted, or Forbidden.

    2. If the player in the lead makes a bonus move and an opponent points out the error before another player makes a legal move or challenge, the Mover must return the second cube played on that turn to Resources. The Mover also loses one point.

        Comment

        (a) Players tied for the lead *may* make Bonus moves.

        (b) Players often call "Bonus" and move two cubes simultaneously to Forbidden. If the player did not call "Bonus," he may return either of the two cubes to Resources.

D. In Middle, Junior, and Senior Divisions, no = or C̲ cubes may be played to Forbidden *until four or fewer cubes remain in Resources*.

    Comments

    (a) V̲ and ∧̲ *may* be played to Forbidden.

    (b) Allowing an = or C̲ to be played to Forbidden with four or fewer cubes left in Resources is intended to cover those rare situations where a player would have no choice but to make a Solution possible with one more cube. For example, there may be a color cube and two = or C̲ cubes left in Resources. If the color cube and either Restriction cube is needed for a Solution, then the Mover would have no choice but to play one of the three cubes to Required or Permitted. The exception above allows the player to move either the = or C̲ cube to Forbidden to avoid a Now challenge.

V. Challenging

A. "Whether or not it is your turn, you may challenge another player who has just completed a move" or set the Goal. The only two legal challenges are Now and Impossible.

    1. By challenging *Impossible*, a player claims that no correct Solution can be written regardless of how the cubes remaining in Resources may be played.

        Comment   If the Goal is not a legal configuration (see section **III-B-1**) or the Goal equals a negative number, an opponent should challenge Impossible.

    2. By challenging *Now*, a player claims that a Solution can be written using the cubes on the mat and, if needed, *one* cube from Resources.

        a. A player may challenge Now only if there are at least two cubes in Resources.

            If a player challenges Now with fewer than two cubes in Resources, the challenge is invalid and is set aside. The challenger is also penalized one point. (See section **B** below.)

            Comment   If only one cube remains in Resources and no one challenges Impossible, then a Solution is possible using that one cube. Since the latest Mover had no choice but to play the second-to-last Resource cube to the mat, it is not fair that he be subject to a Now challenge. However, an Impossible challenge could be made.

b. Since a correct Solution must contain at least two cubes, it is illegal to challenge Now after the Goal has been set but before a cube has been played to Required or Permitted.

If a player does so, the "challenge" is set aside, the player is penalized one point, and play continues.

B. A challenge block is placed equidistant from all players. To challenge, a player must pick up the block and say "Now" or "Impossible."

A player who picks up the block and makes an invalid challenge or says nothing is penalized one point, and the challenge is set aside. Examples of invalid challenges are **(a)** challenging yourself (you were the last Mover), **(b)** challenging Now when less than two cubes remain in Resources, and **(c)** challenging Now with no cubes in Required or Permitted. If a player picks up the block, then decides not to challenge (without saying "Now" or "Impossible"), the player accepts a one-point penalty and play continues.

Comments

(a) The main purpose of the block is to determine who is the Challenger in a three-player match when two players wish to challenge at the same time.

(b) Touching the challenge block has no significance. However, players may not keep a hand or finger on, over, or near the block for an extended period of time. (See section **IX-C**.)

(c) A player must not pick up the challenge block for any reason except to challenge. For example, don't pick it up to say "Goal" or to charge illegal procedure.

VI. The Parts of a Solution

A. Set-Name part*:* this part consists of one legal Set-Name. A Set-Name is legal if it specifies a set of cards in the Universe and does not contain any symbol or group of symbols which is undefined in On-Sets.

= and $\underline{C}$ cubes may not be used in the Set-Name of the Solution even if they are in Required. If the Set-Name part of the Solution contains an = or $\underline{C}$ symbol, it is automatically wrong.

Examples of Set-Names   R', G $\underline{U}$ Y, (R $\underline{\cap}$ B) – $\underline{\Lambda}$, ($\underline{V}$ – G)' $\underline{U}$ R

Comments

(a) A Set-Name written on paper may contain pairs of grouping symbols such as parentheses, brackets, or braces even though these do not appear on the cubes. These symbols indicate how the Solution-writer would physically group the cubes if the Solution were actually built with the cubes.

(b) The Solution-writer must not write = Goal after the Set-Name. Doing so makes the Solution automatically incorrect.

B. Restriction part*:* this part consists of one or more Restrictions.

1. A *Restriction* is a rule that is applied to the cards in the Universe. Any card which does not satisfy the Restriction is temporarily removed from the Universe while that Solution is being checked. After all Restrictions in the Solution have been applied to the Universe, the Solution-writer's Set-Name is worked out using the cards that remain in the restricted Universe.

Comments

(a) Any cards removed while checking a Solution are returned to the Universe before another Solution is checked.

(b) Any Restrictions in a Solution apply to the Universe only for *that* Solution.

2. There are three types of Restrictions. Any set used in each type must be represented by a legal Set-Name.

a. *Subset Restriction*: this type has the form *Set 1* $\underline{C}$ *Set 2*. A card in the Universe does not satisfy a subset Restriction if it is in *Set 1* but not in *Set 2*.

Examples B $\underline{C}$ R', G – Y $\underline{C}$ ∧, B – R $\underline{C}$ (G ∩ V)', B $\underline{U}$ Y $\underline{C}$ B $\underline{U}$ Y

b. *Equals Restriction*: this type has the form *Set 1 = Set 2*. A card in the Universe does not satisfy an equals Restriction if it is in one of the two sets but not in the other.

Examples B = R, G – Y = $\underline{V}$, (B ∩ G)' = Y – R, R = R

c. *Chain Restriction*: this type has two or more = or $\underline{C}$ cubes in it.

   (i) Restrictions of the following form are defined, where *A*, *B*, and *C* are sets.

              A $\underline{C}$ *B* $\underline{C}$ *C*   (meaning *A* $\underline{C}$ *B* and *B* $\underline{C}$ *C*)
              A = *B* = *C*   (meaning *A* = *B* and *B* = *C*)
              A $\underline{C}$ *B* = *C*   (meaning *A* $\underline{C}$ *B* and *B* = *C*)
              A = *B* $\underline{C}$ *C*   (meaning *A* = *B* and *B* $\underline{C}$ *C*)

   (ii) Restrictions of the following form are also permitted, where each one is worked out from left to right like those above.

     A $\underline{C}$ *B* $\underline{C}$ *C* $\underline{C}$ *D*, *A* = *B* = *C* = *D*, *A* $\underline{C}$ *B* $\underline{C}$ *C* = *D*, *A* = *B* $\underline{C}$ *C* $\underline{C}$ *D*, and so on.

3. In a Restriction, no pair of parentheses (or other grouping symbols) may enclose an = or $\underline{C}$ symbol. However, a player may put parentheses around the entire Restriction, like this: (B $\underline{C}$ R $\underline{U}$ G). Or around one side of a Restriction, like this: B $\underline{C}$ (R $\underline{U}$ G). While useless, these parentheses do not make the Restriction wrong.

Comment   A common error is putting parentheses around part of a chain Restriction, like this where *A*, *B*, and *C* are sets: (*A* $\underline{C}$ *B*) $\underline{C}$ *C*, *A* = (*B* = *C*), and so on. Such parentheses make the chain meaningless just as the parentheses in the algebraic equation (2x=3)+7 make it meaningless. Also these parentheses are inappropriate: (B = R)' However, this does not mean that parentheses may not be used at all in Restrictions. Parentheses may legitimately be placed within any Set-Name in a Restriction, as in the following examples.

     (R $\underline{U}$ B) – G = $\underline{V}$, B = (G $\underline{U}$ R)' $\underline{C}$ $\underline{V}$, R' = B $\underline{C}$ (R – Y) $\underline{U}$ $\underline{V}$

Notice in these examples that no pair of parentheses encloses an = or $\underline{C}$.

VII. Writing and Checking Solutions

  A. After a valid challenge, at least one player must write a Solution.

    1. After a Now challenge,

- the Challenger *must* present a Solution.
- the Mover may *not* present a Solution.
- the Third Party *may* present a Solution.

    2. After an Impossible challenge,

- the Challenger may *not* present a Solution.
- the Mover *must* present a Solution.
- the Third Party *may* present a Solution.

  B. To be *correct*, a Solution must satisfy the following criteria.

    1. The Solution contains a valid Set-Name part.

    2. Middle, Junior, Senior only: The Solution contains a Restriction part if there are one or more = or $\underline{C}$ cubes in Required.

If no = or $\underline{C}$ cubes are in Required but some are in Permitted or Resources, the Solution *may* include a Restriction part.

3. The Solution equals the Goal. That is, the number of cards selected from the Universe by the Set-Name equals the Goal.

    If the Solution includes one or more Restrictions, these must be applied to the Universe *before* the Set-Name is worked out. If there are two or more Restrictions, they may be applied to the Universe in any order.

    Comments

    (a) Unlike *Equations*, the Solution-writer must *not* write = Goal after the Set-Name.
    (b) Mid/Jr/Sr: With the Absolute Value variation (see section **XIII** below), the Goal may have be ambiguous. Then any Solution must equal one of the legal values of the Goal.

4. The Solution uses the cubes correctly.

    a. The Solution contains at least *two* cubes.

    Example   In Middle, Junior, and Senior Divisions, the following Solution satisfies this rule:

    Restriction: B = B
    Set-Name: B

    The Solution contains three cubes even though the Set-Name contains only one.

    b. Every cube in Required is used in the Restriction part (if there is one). These same cubes (except any = or C̲) must also be used in the Set-Name.

    c. Each cube in Permitted may be used in the Restriction part (if there is one). These same cubes (except any = or C̲) may also be used in the Set-Name.

    d. The Solution uses *no* cube in Forbidden.

    Comment   "Since several Resource cubes may show the same symbol, it is possible to have a U̲ in Forbidden which *must not* be used in the Solution at the same time that there is a U̲ in Required which *must* be used."

    e. After a Now challenge, the Solution may contain *at most one* cube from Resources.

    f. After an Impossible challenge, any cubes in Resources are considered to be in Permitted and therefore may be used in the Solution.

5. In Adventurous On-Sets, the Solution satisfies all conditions imposed by the variations selected for that shake. (See section **XIII** for the list of variations.)

    Examples

    (a) If the variation "Required –" has been chosen, each Solution must contain a – sign.
    (b) If the Two Operations variation has been chosen, then the Set-Name part of every Solution must contain at least two operation symbols.

6. Every legal interpretation of the Solution equals the Goal.

    a. An *ambiguous Solution* is one that has more than one legal interpretation. Such a Solution is incorrect if an opponent shows that one of the interpretations does not equal the Goal.

    b. The only defined order of operations in On-Sets is that the ' operation takes priority over all other operations (U̲, ∩, –, and special operations defined by variations). Consequently, a Solution may be ambiguous if the writer does not use parentheses (or other grouping symbols such as brackets or braces) to indicate the order of operations.

C. After the time for writing Solutions has expired (or when all Solution-writers are ready), each Solution that is presented must be checked for correctness.

   1. After a challenge in a three-player match, the Third Party has two minutes to decide whether or not to present a Solution.

Comment   The Third Party is not obligated to indicate whether he is presenting a Solution before the time limit expires. The Third Party may indicate his decision by:
(a)   stating whether or not he is presenting a Solution;
(b)   answering "yes" (verbally or with a nod) or "no" (verbally or with a shake of the head) when asked whether he is presenting;
(c)   Handing his paper to an opponent for checking.

Once the Third Party has indicated whether or not she is presenting a Solution, she may not retract her decision even if the time for presenting Solutions has not expired.

2. All Solutions must be presented before any is checked.

   a.   Once a player presents a Solution to the opponent(s), she may make no further corrections or additions even if the time for writing Solutions has not expired. If the writer tries to make a change after submitting his Solution, his Solution is automatically incorrect.

   b.   Each Solution-writer must circle the Solution to be checked. A writer who forgets to circle the Solution must do so immediately when asked by an opponent.

3. Opponents have two minutes to check each Solution. When more than one Solution must be checked, they may be checked in any order. In a three-player match, *both* opponents must check a player's Solution during the *same* two minutes. No other Solution should be checked during this time.

   Comment   When both players in a two-way match present Solutions after the last cube has been moved (see section **VIII** below), only one Solution should be checked at a time.

4. Within the time for checking a Solution, opponents must accept or reject the Solution. If the Solution is rejected, an opponent must show that it violates at least one of the criteria in section **VII-B**. A Solution is correct if no opponent shows that it is incorrect.

   After a challenge in a three-player match, a player who does not present a Solution for a shake scores 2 if he accepts another player's Solution as correct even if that Solution is subsequently proved wrong by the other checker.

   Comment   Players must not use the cubes on the playing mat to form the Solution since this causes arguments over where each cube was played on the mat.

5. A player who claims an opponent's Solution does not equal the Goal must give at least one of the following reasons.

   a.   The Goal has no legal interpretation.
      Examples
      (a) The Goal is in the shape of a backwards L, which is not a legal configuration.
      (b) The Goal equals a negative number.

   b.   The Solution equals a value which is not a legal value of the Goal. (The only time when the Goal might have more than one value would be in Mid/Jr/Sr when the Absolute Value variation is in effect – see section **XIIIB**.)

      (i)   Checkers must *make an effort* to determine whether the Solution equals the Goal before rejecting a Solution. This can usually be done by applying the Solution to the Universe and turning over cards (if they disobey the Restriction) and/or selecting out the cards that are included in the Set-Name.

      (ii) The checker can give a general argument that the Solution does not equal the Goal.
         Examples
         (a) The Goal is 0, and the Solution clearly does not give the null set.
         (b) Mid/Jr/Sr: The Goal is 5, and the Restriction removes all but 4 (or fewer) cards in the Universe (with no Double Set in Jr/Sr).

c. The Solution may be grouped so that it does not equal any value of the Goal. If a checker believes there is an interpretation of a Solution which does not equal the Goal, that checker must copy the Solution on his paper and add grouping symbols to create a *wrong* interpretation. If there is a second checker, the checkers may either work together to prove ambiguity or work separately. If working separately, the second checker may simultaneously and independently try to prove ambiguity. When both checkers are ready (or one is ready and the other has nothing to show), follow the same procedure used for checking the original Solution. That is, each attempt at proving ambiguity is checked. If either shows a legal interpretation of the Solution such that the Solution does not equal the Goal, then the original Solution is incorrect. If each attempt at proving ambiguity fails, the Solution must be accepted as correct. That is, *once the Solution-writer starts checking the attempt(s) at proving ambiguity, no further objections to the Solution are allowed.*

Comments
(a) In the case where the checkers work separately to prove ambiguity, if the time for checking the Solution runs out, either or both checkers may take an additional minute (paying the one-point penalty to do so). If only one checker wishes to take the additional minute, the other checker may make no further changes to his revision of the Solution. If he tries to do so, then he incurs the one-point penalty also.
(b) Just as two players writing Solution after a challenge or forceout may not communicate with each other, so two checkers attempting to prove ambiguity separately may not communicate while doing so.
(c) Each checker working separately has only *one* opportunity to prove ambiguity. Similarly, checkers working together have just one joint chance to prove ambiguity.
(d) While only one checker is attempting to prove ambiguity, the other checker may continue to check other aspects of the Solution.
(e) If each checker separately trying to prove ambiguity is ready with his revision of the Solution before the time for checking expires, no -1 penalty is enforced during the time the original Solution-writer checks any attempts at proving ambiguity.

Examples
 (a) The Set-Name B U̲ G – R is ambiguous and may be interpreted by an opponent as (B U̲ G) – R or as B U̲ (G – R). If the interpretation the opponent selects does not equal the Goal, the Solution is incorrect.
(b) R U̲ G' is *not* ambiguous. It must be interpreted as R U̲ (G') since ' takes priority over U̲.
Comment   Some variations (such as Y Wild) allow certain cubes to be used for other symbols. If a Solution-writer wishes a cube to stand for anything other than what is on the cube, she must indicate clearly and unambiguously in writing what each such cube represents. (See Appendix **A** for a list of suggested ways of doing this.)

d. A symbol or group of symbols in the Solution has no defined meaning.
Examples

(a) The Set-Name is R U 'B or R /\ G.
(b) Mid/Jr/Sr: The Restriction is R U̲ (B = G) – R.

e. A variation is applied incorrectly or not at all.
Examples of incorrect Solutions

(a) With – Wild, a Solution uses a – for one symbol and another – for a different symbol.
(b) With Two Operations, the Solution contains only one operation.

6. One or both of the checkers may ask a judge to determine whether the Solution equals the Goal.

a. *No further objections to the Solution will be allowed* even if the time limit for checking has not expired.

    b. If the Solution is ambiguous, the judge will answer, "Yes, the Solution equals the Goal" when a value of the Solution equals the Goal (or one of the values of the Goal if Absolute Value is in effect) since the checkers did not raise the issue of ambiguity.

  7. A player may appeal a judge's ruling on any matter provided that **(a)** a second judge was called to rule on the situation and **(b)** the player does not initial the scoresheet at the end of the round.

> Comment   For the procedure to be followed when a player appeals, see the *National Tournament Administrative Book*.

VIII. Last Cube Procedure

A. If one cube remains in Resources, the next Mover must either play that cube to Required or Permitted or challenge Impossible. When the cube has been moved, each player has two minutes to write a Solution.

> The last cube in Resources may *not* be moved to Forbidden. If a player does so, any challenge that is made is set aside and the cube is returned to Resources. There is no penalty for the move to Forbidden unless the player's time to move or challenge Impossible expires. (See section **XI**.)

B. An opponent may challenge Impossible against the player who moved the last cube provided the challenge is made by the end of the first minute for writing Solutions. If the challenge is made, any Solution-writer has the rest of the original two minutes to write a Solution.

> *Comment*   Any Now challenge against the player moving the last cube is invalid as is any Impossible challenge made after the first minute for writing Solutions. In both cases, the player attempting to challenge loses a point, and the challenge is set aside.

IX. Illegal Procedures

A. Any action which violates a procedural rule is illegal procedure. A player charging illegal procedure must clearly specify immediately the exact nature of the illegal procedure.

  1. If a move *is* illegal procedure, the Mover must return any illegally moved cube(s) to their previous position (usually Resources) and, if necessary, make another move.

> The Mover must be given at least 10 seconds to make this correction, unless the original move was made after the ten-second countdown (see section **XI-A-3** below), in which case the time limit rule (section **XI-A**) is enforced. In general, there is no direct penalty except that the Mover may lose a point if she does not legally complete her turn during the time limit.
>
> *Examples of illegal procedures*
>
> Moving out of turn, moving two cubes without calling "Bonus" before the first cube touches the mat in Forbidden, moving the last cube in Resources to Forbidden, or (in Middle, Junior, and Senior) moving = or C̲ to Forbidden with more than four cubes left in Resources.

  2. If the move is *not* illegal procedure, the cube stands as played.

> *Comment*   There is no penalty for erroneously charging illegal procedure. However, see section **C** below if a player does so frequently.

B. An illegal procedure is insulated by a legal action (for example, a move or challenge) by another player so that, if the illegal procedure is not corrected before another player takes a legitimate action, it stands as completed.

> *Example*   Suppose the player in the lead makes a bonus move. Before anyone notices the illegal procedure, the next mover moves (or a valid challenge is issued). Then the illegal bonus move stays in Forbidden without penalty.

C. Certain forms of behavior interfere with play and annoy or intimidate opponents. If a player is guilty of such conduct, a judge will warn the player to discontinue the offensive behavior. Thereafter during that round or subsequent rounds, if the player again behaves in an offensive manner, the player may be penalized one point for each violation after the warning. Flagrant misconduct or continued misbehavior may cause the player's disqualification for that round or all subsequent rounds. Judges may even decide to have the other two opponents replay one or more shakes or the entire round because play was so disrupted by the third party. In some cases, judges may order the shake replayed by all three players.

Examples   This rule applies to use of a cell phone, constant talking, tapping on the table, humming or singing, loud or rude language, keeping a hand or finger over or next to the challenge block, making numerous false accusations of illegal procedure, and so on. It also includes not playing to win but rather trying only to ruin the perfect scores of one or both opponents (for example, by erroneously challenging Now or Impossible at or near the beginning of each shake so that both opponents will score 5 for the round), saying one variation but circling another, counting down the ten-second warning in an obnoxious manner, etc.

D. Certain infractions that give a player an unfair advantage or completely disrupt a shake may draw a -1 penalty immediately without a warning provided at least two judges agree on the penalty. Examples include: Consulting notes that were written before the match began; rerolling the cubes after they were legally rolled; holding the cards face up when dealing; re-dealing some or all the cards; intentionally turning over a cube on the playing mat or in Resources; hiding a card in the Universe or adding a card to the Universe during the shake; and saying one variation selection but circling another. A pair of judges may also issue a -1 penalty or even expel a player from a match for other egregious actions such as not playing to win but rather trying only to ruin the perfect scores of one or both opponents (for example, by erroneously challenging Now or Impossible at or near the beginning of each shake so that both opponents will score 5 for the round), knocking cubes off the mat in a fit of pique before the shake is finished, intimidating an opponent verbally or with threatening gestures or body language, refusing to continue play when ordered by a judge, and so on.

X.   Scoring a Shake

A. After a challenge, a player is *correct* according to the following criteria.

1. That player had to write a Solution and did so correctly.

If the Third Party agrees with the person who must write a Solution, the Third Party must write a correct Solution also.

2. That player did not have to write a Solution (someone else did), and no opponent wrote a correct Solution.

Exception: After a Challenge in a three-player match, a player who does not present a Solution for a shake scores 2 if he accepts another player's Solution as correct even if that Solution is subsequently proved wrong by the other checker.

B. After a challenge, points are awarded as follows.

1. Any player who is not correct scores 2.

A player is not correct if the player:

- presented an incorrect Solution.
- challenged Impossible, and an opponent presented a correct Solution.
- as Third Party on a Now challenge, did not present a Solution, but the Challenger did present a correct Solution.
- as Third Party on an Impossible challenge, did not present a Solution, but the Mover did present a correct Solution.

2. A correct Challenger or Mover scores 6.

3. The Third Party scores 6 if that player:

- presented a correct Solution after an Impossible challenge;
- presented a correct Solution after a Now challenge and the Challenger did *not* present a correct Solution.

- did not have to write an Equation (someone else did), and no opponent wrote a correct Equation.

    4. The Third Party scores 4 if that player:

- did not present a Solution after an Impossible challenge and the Mover did not present a correct Solution.
- presented a correct Solution after a Now challenge and the Challenger also presented a correct Solution.

C. After the last cube from Resources is moved to the playing mat and no one challenges Impossible, points are awarded as follows.

    1. Any player who writes a correct Solution scores 4.

    2. Any player who does not write a correct Solution scores 2.

D. A player who is absent for a shake scores -2 for that shake.

XI. Time Limits

A. Each task a player must complete has a specific time limit as listed below. The one- and two-minute time limits are enforced with the timer. If a player fails to meet a deadline, he loses one point and has one more minute to complete the task. If he is not finished at the end of this additional minute, another one-point penalty is imposed, and he loses his turn or is not allowed to complete the task.

Note: In Elementary and Middle Divisions, each one-point penalty (for whatever reason) must be approved (initialed) by a judge on the scoresheet.

    1. The time limits are as follows.

a. rolling the cubes	1 minute
b. making a variation selection	15 seconds

        This time limit does not begin until after the one minute for rolling the cubes.

c. setting the Goal	2 minutes
d. first turn of the player to the left of the Goal-setter	2 minutes
e. all other regular turns (including any bonus moves)	1 minute
f. stating a valid challenge after picking up the challenge block	15 seconds
g. deciding whether to challenge Impossible when no more cubes remain in Resources	1 minute

        If the Impossible challenge is made, any time (up to a minute) the Challenger takes deciding to challenge counts as part of the two minutes for writing a Solution.

h. writing a Solution	2 minutes

        During this time, the Third Party (if there is one) must decide whether to present a Solution after a Now or Impossible challenge.

i. deciding whether an opponent's Solution is correct	2 minutes

    2. Often a player completes a task before the time limit expires. When sand remains in the timer from the previous time limit, the next player will receive additional time. An opponent timing the next player may either flip or not flip the timer so as to give the opponent the *lesser* amount of time before the remaining sand runs out, and the next time limit can be started.

    3. A player who does not complete a task before sand runs out for the time limit must be warned that time is up. An opponent must then count down 10 seconds loud enough for the opponent to hear. The one-point penalty for exceeding a time limit can be imposed only if the player does not complete the required task by the end of the countdown.

The countdown must be done at a reasonable pace; for example, "1010, 1009, ..., zero."

An exception to this rule occurs when a player picks up the Challenge Block but does not state a valid challenge within the 15 second time limit. If the player does not wish to challenge, he loses one point and play continues.

B. Each round lasts 30 minutes. When that time is up, players are told not to start any more shakes. Any shake for which there has been no challenge and the last cube procedure is not underway continues as follows.

1. Players have five minutes to finish the last shake.

2. When the extra five minutes expire, players still involved in a shake in which no challenge has been made and one or more cubes remain in Resources will be told: "Stop; do not play another cube to the mat or make a challenge. Each player has two minutes to write a correct Solution that may use *any* of the cubes remaining in Resources." Any player who presents a correct Solution scores 4 points for that shake; a player who does not present a correct Solution scores 2.

Comment In Senior Division shakes where Two Solutions is in force, players have three minutes to write Solutions.

XII. Scoring a Match

A. Each player is awarded points for the match based on the sum of his scores for the shakes played during that match according to the following tables.

Three-Player Matches	Points
first place	6
two-way tie for first	5
three-way tie for first	4
second place	4
tie for second	3
third place	2

Two-Player Matches	Points
first place	6
two-way tie for first	5
second place	4

B. When a round ends, each player must sign (or initial) the scoresheet and the winner (or one of those tied for first) turns it in. If a player signs or initials a scoresheet on which his score is listed incorrectly and there is evidence that there was intent to deceive and the error was not a simple oversight, then do the following.

1. If the error gives the player a lower score, he receives the lower score.

2. If the error gives the player a higher score, he receives 0 for that round.

XIII. Adventurous Variations

Comment *See Section **II-B** for the procedure to be followed when selecting variations.*

A. Elementary Variations (grade **6** and below)

1. Required Cube The Solution must contain a ____ cube. The player selecting this variation specifies which non-digit symbol from the Resources fills the blank in the previous sentence.

Comment If, say, Required – is chosen along with B Wild, a B cube used as – does *not* satisfy the Required Cube variation.

2. Wild Cube The __ cube may represent any symbol on the cubes except a digit. The __ cube must stand for the same symbol everywhere it occurs in the Solution. The player selecting this variation specifies which cube from the Resources is wild. The wild cube may not be a digit. Each Solution-writer must specify in

writing the interpretation of the wild cube if it stands for anything other than itself in his Solution.

Comments

(a) If both B Wild and B Required are chosen, a B *cube* must be in the Solution but may stand for another symbol.

(b) See Appendix **A** for examples of ways to indicate what a wild cube stands for in a Solution. However, if B is wild but used as B, this need not be indicated.

3. U and ∩ Interchangeable Any U may represent U or ∩, and any ∩ may represent ∩ or U.

Comments

(a) U and ∩ need not be used consistently. In a Solution, one U (or ∩) may be used as U and another U (or ∩) used as ∩.

(b) Any wild cube used as U or ∩ gains the full interchangeable power granted U and ∩ by this variation.

(c) If U (or ∩) Wild and U-∩ Interchangeable are both chosen for a shake, then, if U (or ∩) is used just for itself or ∩, it need not be used consistently. However, if U (or ∩) is used for any symbol other than U or ∩, then it must represent that same symbol throughout the Solution.

(d) Since this variation makes U and ∩ "wild" in only a limited way, players are *not* required to indicate in writing where in the Solution a U stands for ∩ or a ∩ stands for U. They should simply write the symbol they want mathematically.

(e) If U Wild is also called, this does not mean ∩ cubes are wild and vice-versa.

4. V and ∧ Interchangeable Any V may represent V or ∧, and any ∧ may represent ∧ or V.

Comment The comments above for U and ∩ Interchangeable, substituting V for U and ∧ for ∩, apply here.

5. Two Operations Each Solution must contain at least two operation symbols. The operation symbols are U, ∩, −, and '.

Comments

(a) If a wild cube is also chosen, a wild cube used as an operation counts as an operation *symbol*. On the other hand, any wild operation cube not used as an operation does *not* count as an operation symbol.

(b) A Solution like R U B U V satisfies this variation. The variation does not require two *different* operation symbols in the Solution.

6. Multiple Operations Any operation sign not in Forbidden may be used many times in any Solution.

Comments

(a) After an Impossible challenge, any operation sign in Resources may be used many times in any Solution. After a Now challenge, if the one cube allowed from Resources is an operation cube (or a wild cube used as an operation), it may be used multiple times.

(b) With this variation, an operation cube is not used to represent another symbol. So players may simply write an operation sign multiple times in Solutions without any additional indication.

7. Shift from Permitted On your turn, you may transfer a cube in Permitted to either Required or Forbidden. This move takes the place of your regular move.

Comments

(a) If not in the lead, you may make a bonus move from Resources to Forbidden before transferring a cube out of Permitted as your regular move.

(b) You may *never* shift a cube from Permitted to Forbidden as a Bonus move.

(c) Once the last cube in Resources has been moved to Required or Permitted, no more cubes from Permitted may be shifted.

B. Middle Variations (grade **8** and below)

The following Elementary variations may also be chosen in Middle. (See the comments following each in the Elementary list in addition to any comments below.)

1. <u>Required Cube</u> The Solution must contain a ___ cube. The player selecting this variation specifies which non-digit symbol from the Resources fills the blank in the previous sentence.

   Comment   If a player selects = or <u>C</u> Required, this variation is satisfied by using the required cube in a Restriction. If the required cube is a color, <u>V</u> or <u>∧</u>, or an operation symbol, the variation is satisfied by using that symbol in *either* a Restriction or the Set-Name. However, in the latter case, if the required symbol is played to Required, then, as usual, it must be in *both* a Restriction (if one is made) and the Set-Name.

2. <u>Wild Cube</u> The __ cube may represent any symbol on the cubes except a digit. The __ cube must stand for the same symbol everywhere it occurs (Restriction(s) and Set-Name). The player selecting this variation specifies which cube from the Resources is wild. The wild cube may not be =, <u>C</u>, or a digit. Each Solution-writer must specify in writing the interpretation of the wild cube if it stands for anything other than itself in his Solution.

3. <u>U and ∩ Interchangeable</u> Any <u>U</u> may represent <u>U</u> or <u>∩</u>, and any <u>∩</u> may represent <u>∩</u> or <u>U</u>.

4. <u>V and ∧ Interchangeable</u> Any <u>V</u> may represent <u>V</u> or <u>∧</u>, and any <u>∧</u> may represent <u>∧</u> or <u>V</u>.

5. <u>Two Operations</u> The Set-Name of each Solution must contain at least two operation symbols. The operation symbols are <u>U</u>, <u>∩</u>, −, and '.

6. <u>Multiple Operations</u> Any operation sign not in Forbidden may be used many times in a Solution (Set-Name or Restriction or both).

7. <u>Shift from Permitted</u> On your turn, you may transfer a cube in Permitted to either Required or Forbidden. This move takes the place of your regular move.

   Comment   You may never shift an = or <u>C</u> cube from Permitted to Forbidden (even when there are four or fewer cubes in Resources).

Middle Division players may also choose the following variations.

8. <u>No Null Restrictions</u> Each Restriction must remove at least one card from the Universe. In a chain Restriction, this variation is satisfied if any part of the Restriction removes a card.

   Comment   If a Solution includes more than one Restriction, each must remove at least one card regardless of the order in which they are applied to the Universe.

9. <u>Absolute Value</u> Any upside-down cube(s) in the Goal may be interpreted as right-side-up by a Solution-writer.

   Examples

   (a) The Goal 3⅃ (upside-down 2) may be interpreted as 1 or 5.

   (b) The Goal ⅃Ⱶ (where the 2 and 1 are upside-down) may equal 5 or 7. The 2 *must* be interpreted as right-side up in order to create a legal (positive) value.

C. Junior Variations (grade **10** and below)

SPECIAL RULE: The following three variations are in effect for *all* shakes.

1. <u>Multiple Operations</u> Any operation sign not in Forbidden may be used many times in any Solution (Set-Name or Restriction or both).

2. <u>U and ∩ Interchangeable</u> Any <u>U</u> may represent <u>U</u> or <u>∩</u>, and any <u>∩</u> may represent <u>∩</u> or <u>U</u>.

3. <u>V and ∧ Interchangeable</u> Any <u>V</u> may represent <u>V</u> or <u>∧</u>, and any <u>∧</u> may represent <u>∧</u> or <u>V</u>.

Junior players may choose any of the following Middle variations. (See the comments and examples after each variation in the Elementary and Middle lists.)

4. <u>Required Cube</u> The Solution must contain a ___ cube. The player selecting this variation specifies which non-digit symbol from the Resources fills the blank in the previous sentence.

5. <u>Wild Cube</u> The __ cube may represent any symbol on the cubes except a digit. The __ cube must stand for the same symbol everywhere it occurs (Restriction(s) and Set-Name). The player selecting this variation specifies which cube from the Resources is wild. The wild cube may not be =, <u>C</u>, or a digit. Each Solution-writer must specify in writing the interpretation of the wild cube if it stands for anything other than itself in his Solution.

6. <u>Two Operations</u> The Set-Name of each Solution must contain at least two operation symbols. The operation symbols are <u>U</u>, <u>∩</u>, –, and '.

7. <u>Shift from Permitted</u> On your turn, you may transfer a cube in Permitted to either Required or Forbidden. This move takes the place of your regular move.

8. <u>No Null Restrictions</u> Each Restriction must remove at least one card from the Universe. In a chain Restriction, this variation is satisfied if any part of the Restriction removes a card.

9. <u>Absolute Value</u> Any upside-down cube(s) in the Goal may be interpreted as right-side-up by Solution-writers.

Junior Division players may also choose the following variations.

10. <u>Double Set</u> Each card in the Universe that is contained in the ___ set will count double for all Solutions. The player selecting this variation specifies which non-empty set of cards *that does not equal the Universe* counts double. The set must be named using an expression consisting of *at most four* symbols (not counting grouping symbols).

    Examples

    A player selecting this variation may choose to double B, R', G ∩ Y, (B – R)', <u>V</u> – B', and so on. Players may not specify Double Sets like B <u>U</u> R <u>U</u> G, Y – (B ∩ G)', B' – R', <u>V</u> – (R – B), and so on. Also if a player selects R – Y as the doubled set but there are no cards in R – Y, the player is penalized one point and must select another variation. Similarly, if a player selects B <u>U</u> R' as the Double Set and every card in the Universe is B or R', the player loses a point and must pick another variation.

    Comment   In Senior Division, if a player specifies a Double Set using –, that – means regular subtraction even if a *subsequent* player (or the same player) calls Symmetric Difference. However, if Symmetric Difference is called first, then any – in a Double Set called by a subsequent player (or the same player) means Symmetric Difference.

11. Required/Forbidden Card The player selecting this variation either specifies one card in the Universe which must be in the Set-Name of any Solution or specifies one card in the Universe which must *not* be in the Set-Name of any Solution.

Comments

(a) The player states the required or forbidden card orally and records the card in a blank on the variation selection sheet. For example, "BRG", "RY", "blank", and so on.

(b) Suppose Blank Card Wild (see below) is chosen along with, say "BR forbidden." If the blank card is made the forbidden card (BR) for a Solution, then neither the blank card nor the BR card may be part of the Set-Name of the Solution.

12. Blank Card Wild Each Solution-writer must specify in writing which colors, if any, are on the blank card.

Comments

(a) This variation may be chosen only if the blank card has been dealt.

(b) If the blank card remains blank for a Solution, the Solution-writer does not need to specify this.

(c) Suppose Double Set and Blank Card Wild are both chosen with, say, B the Double Set. If a player chooses to put a B dot on the blank card, the blank card counts double for that player's Solution.

(d) If Required Card is also chosen with the blank card required, that variation is satisfied if the blank card is in the Set-Name no matter if the Solution-writer puts one or more colors on the blank card.

(e) Blank Card Wild and Blank Card Forbidden are in conflict. So if Blank Card Wild is selected, a player then choosing Blank Card Forbidden is penalized one point and vice-versa.

D. **Senior** Variations (grade **12** and below)

Players may choose any of the Junior variations (except for the three which are in effect for every shake) plus the following.

13. Symmetric Difference The – symbol means symmetric difference; that is, $A - B$ equals $(A - B) \cup (B - A)$, where these last two – signs mean set subtraction.

If – Wild has been selected for the shake, no player may select Symmetric Difference for that shake. Similarly, if Symmetric Difference has been chosen, no player may select – Wild. (In either case, the player selecting the second of the two conflicting variations receives a -1 penalty.)

Comments

(a) If Wild Cube is also chosen, any wild cube used as – means symmetric difference, not set subtraction.

(b) In Solutions, players simply write the – sign with the understanding that it means symmetric difference.

(c) **When both Symmetric Difference and Double Set are chosen for a shake by different players, players must indicate on the Variation Selection Sheet the *order* in which the two variations were chosen. Write a 1 next to the one circled first and a 2 next to the other. If both variations were chosen by the same player, any – in the Double Set means set subtraction, not Symmetric Difference.**

14. Two Solutions Each Solution-writer must write two Solutions; the set named by the second Solution must contain at least one card that is not in the set named by the first Solution.

When this variation is in effect, players have *three* minutes to write Solutions and *three* minutes to check each player's pair of Solutions. Also no Now challenges may be made with fewer than *three* cubes left in Resources. Furthermore, when the Mover plays the last cube from Resources to Required or Permitted, the other players have *two* minutes to challenge Impossible.

Comments

(a) An Impossible challenge should be made against a Goal of 0 since it is impossible to satisfy the Two Solutions variation in this case. Similarly, a Goal equal to the number of cards in the

Universe is impossible. With Double Set, the Goal can legitimately be larger than the number of cards. However, if the Goal is such that all cards in the Universe must be in any Solution, then an Impossible challenge should be made.

(b) In determining the rules a player's two Solutions must follow, it is helpful to think of the Solutions as if they were presented by *different* players. Each Solution must use the cubes correctly and obey all the variations for the shake.

(c) After a Now challenge, Solution A of a player may use one cube from Resources and Solution B of that player may use a *different* Resource cube (or no cube).

(d) A wild cube may stand for one symbol throughout one of a player's Solutions and another symbol throughout the other Solution.

(e) With Blank Card Wild, a Solution-writer may put one set of dots (or no dots) on the blank card for one Solution and another set of dots on the blank card for the other Solution. However putting a different set of dots on the blank card does not make it a different card for the second Solution. Thus, if the first Solution yields a set consisting of cards 1, 2, 3 and the blank card and the second Solution produces cards 1, 2, 3, and the blank card (with a different set of colors on it), these Solutions do not satisfy this variation. Similarly, if the Goal is 1, a player may not write one Solution that produces the blank card with a certain set of dots on it (or no dots) and a second Solution that produces the blank card with a different set of dots on it.

(f) Suppose Required Card is chosen, and the Goal is 1. An opponent should challenge Impossible since it is impossible to write two Solutions that produce a different card and satisfy the Required Card variation.

(g) With Absolute Value, one Solution may equal one interpretation of the Goal, and the other Solution may equal the other interpretation.

(h) If both Solutions have the same Restriction, the writer must indicate this in an unambiguous manner. (See Appendix A for ways to do so.)

Appendix A: Ways of Indicating What Cubes Mean in Solutions
Some General Principles

1. Each Solution-writer must not only create a correct Solution but must also clearly *communicate* the Solution to the Checker(s) so that they can verify that the Solution equals the Goal.

2. Each Solution-writer must remove all ambiguity from the Solution. Removing ambiguity has two components: **(a)** using grouping symbols to specify the order of operations and **(b)** indicating the value of any cube in the Solution that may have multiple meanings. It is component **(b)** that is the subject of this Appendix, although in some cases placement of parentheses may clarify the meaning of a symbol.

3. In general, a Solution-writer should write in the main line of the Solution the *meaning* of each cube that represents something other than its "face value." For example, write what the wild cube represents in the Solution and indicate above or below that the wild cube is being used. This principle is implemented in the *Recommended* methods in this Appendix. The reverse technique, listing the wild cube in the Solution and indicating its meaning from the side, is *Acceptable* only.

4. If the Soution-writer does a good job, the Checker(s) should not have to ask a single question about the Solution. It should be clear what each symbol means and which interpretation of the Goal the writer has chosen.

5. In general, arrows are preferable for indicating what a cube means, like this.

<div align="center">

B

↓

R–G

</div>

The arrow can come from above or below and can point to or from the symbol in the Solution. Writing the meaning just above or just below the mainline of the Solution without an arrow is acceptable but has the drawback that the two symbols may overlap and confuse rather than clarify.

Explanation of Terms

Methods of writing entire Solutions or individual symbols in Solutions are divided into three categories in the list in this Appendix: *Recommended*, *Acceptable*, and *Unacceptable*. Here are the intended meanings of these terms.

Recommended   This is the method that should be taught to players.

Acceptable   Any method in this category will be accepted by judges as correct.

Unacceptable   These methods will cause the Solution to be ruled incorrect by judges.

Division	Variations	Examples	Default
All	Wild cube	*Recommended:* R U G, Y', or B – V B **wc** U The arrows may come from the top or not appear at all. *Acceptable:* R B G or B U V U The arrows may come from the top or not appear at all.	Wild cube = itself; it is also sufficient to indicate in *one* place in the Solution what the wild cube represents – it is understood to represent the same symbol throughout (Restriction and Set-Name).
All	U, ∩ interchangeable	*Recommended: Write* the symbol (U or ∩) you want in each place in the Solution; no other indication is necessary. *Acceptable:* Indicate the cube upside down in the same way as for a wild cube (above) or upside down cube in Solutions. *Caution:* Writing the bar (_) on an upside-down symbol can cause confusion. It is best *not* to write the upside-down symbols like this: Ū (for upside-down intersect) or n̄ (for upside-down union).	U = union, ∩ = intersection
All	V, ∧ interchangeable	Same as for U, ∩ interchangeable with the same caution.	V = Universe, ∧ = empty set
All	Multiple Operations	*Recommended:* Write the operation sign as many times as you want; no special indication of mult. op. is necessary.	
JS	Blank Card Wild	*Recommended:* Draw a picture of the blank card with the colors indicated, like this: B G *Acceptable:* Blank = BG	Blank card stays blank.

SAMPLE CHARTS FOR RECORDING
THE ON-SETS UNIVERSE

1	2	3	4	5	6	7	8	9	10	11	12	13	14

(Delete #13-14 for divisions other than Senior.)

B R G Y	B R G	B R Y	B R	B G Y	B G	B Y	B	R G Y	R G	R Y	R	G Y	G	Y	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

Each player crosses out the cards that were *not* dealt.

	-	B	BR	R
-				
G				
GY				
Y				

In these last three charts, the color letters may be in a different order or not preprinted at all.
The important criterion is that NO SETS MAY BE PRESHADED OR INDICATED IN ANY
WAY WITHIN THE 16 CELLS OF THE CHART THAT REPRESENT THE CARDS.

Glossary of Terms for On-Sets®

Additional minute	The extra 60 seconds a player receives to complete what he must do after the first time limit has expired; by the end of this extra minute, the player must complete the action or receive a second -1 and forfeit what he is doing
Advanced On-Sets	The form of On-Sets in which Restrictions are used; Middle, Junior, and Senior Divisions play Advanced On-Sets
Adventurous On-Sets	The form of On-Sets in which players choose variations at the beginning of each shake; all four divisions play this form of the game
Adventurous variation	One of a list of special rules used in On-Sets; for each shake each player must select one or two rules from the list for the appropriate division
Ambiguous Goal	A Goal that has more than one legal interpretation; this can happen only when the Absolute value variation is in effect in Middle, Junior, or Senior division
Ambiguous Solution	A Solution that has more than one legal interpretation
Basic On-Sets	The form of On-Sets in which neither Restrictions nor adventurous variations are used
Blank card	The On-Sets card that has no color dots on it
"Bonus"	The word a Mover must say before making a bonus move
Bonus move	A move to Forbidden which a player makes after saying the word "Bonus" and before making another move to any one of the three sections of the mat
Chain Restriction	An On-Sets Restriction that contains two or more = or C symbols
Challenge	A claim by an opponent that a Solution is possible Now or that it is Impossible
Challenge block	An object that a Challenger must pick up in order to challenge
Challenger	The player who makes a legal challenge
Clockwise	To the left
Color cube	A cube with a color dot on each of its faces
Complement of a set	The set of cards in the Universe that are not in the set; this operation is indicated by the symbol '
Correct Solution	A Solution that satisfies all the requirements listed in Section **VII-B** of the On-Sets Tournament Rules

Countdown	The procedure a player uses to tell the Mover that his time is running out; the countdown must be done at a measured pace: "one thousand ten, one thousand nine, ..., zero."
Cube	A block containing a symbol on each of its six faces
Deal cards	Shuffling the cards, then setting out of the required number of cards from the top of the face-down deck in the dealer's hand (that is, the dealer may not select which cards to deal)
Digit	One of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
Digit cube	A cube containing the digit 1 on two of its faces and the digits 2, 3, 4, and 5 on the other four faces
Division	One of the four levels of competition: Elementary (grade 6 and below), Middle (grade 8 and below), Junior (grade 10 and below), and Senior (grade 12 and below)
Elementary Division	The division of play for students in grade 6 and below
Empty set	The set consisting of no cards
Forbidden	The section of the playing mat labeled FORBIDDEN; cubes in this section may not be used in any Solution
Forceout	The term used for the situation in which the last cube from Resources is played to Required or Permitted
Goal	The physical configuration of one or more digit cubes placed on the GOAL section of the playing mat; also any number represented by a legal interpretation of those cubes
"Goal"	The word the Goal-setter says when finished setting the Goal
Goal section	The section of the playing mat where the Goal is placed
Goal-setter	The player who rolls the cubes and sets the Goal
Grouping symbols	Symbols used in pairs (such as parentheses, brackets, and braces) to indicate the order of operations in a Solution
Illegal challenge	After a player picks up the challenge block, **(a)** a claim other than Now or Impossible or a No Goal challenge, **(b)** a Now or Impossible claim by the last Mover (who is therefore challenging himself), **(c)** a Now or Impossible claim before the Goal has been completed, **(d)** a Now claim with no cubes in Required or Permitted, **(e)** a Now claim with fewer than two cubes left in Resources, or **(f)** an Impossible claim after the first minute following the move of the last cube in Resources to Required or Permitted
Illegal procedure	A move or other action that violates a procedural rule; examples: moving out of turn, attempting to make a bonus move without first saying "Bonus"

Illegal Universe	A Universe consisting of too many or too few cards; in Elementary, Middle, and Junior Divisions, the Universe must contain from 6 to 10 cards; in Senior Division, the Universe must contain from 10 to 14 cards
"Impossible"	A word used when making an Impossible challenge
Impossible challenge	A challenge that no correct Solution can be written using all the cubes in Required, none, some, or all the cubes in Permitted, and none, some, or all the cubes in Resources
Incorrect Solution	A Solution that violates one or more stipulations of section **VII-B** of the On-Sets Tournament Rules
Insulation	The process whereby an illegal procedure is protected from correction by a subsequent legal action by another player
Intersection	The operation represented by the symbol $\underline{\Omega}$; the intersection of two sets is the set consisting of the cards contained in *both* sets
Invalid challenge	Same as Illegal challenge
Junior Division	The division of play for students in grade 10 and below
Legal interpretation	An evaluation of the Solution or the Goal that does not violate the rules of mathematics or the game of On-Sets
Legal move	**(a)** A bonus move by the Goal-setter (who is not leading in the match), **(b)** setting a legal Goal by the Goal-setter, or **(c)**, after the Goal has been set, the transfer of a cube from Resources to Required, Permitted, or Forbidden by the player whose turn it is with the exception that the last cube in Resources may not be legally played to Forbidden
Match	A sequence of shakes played continuously by two or three players until a specified time limit has been reached
Middle Division	The division of play for students in grade 8 and below
Minus	A word used for the operation of set subtraction (–)
Minute	The length of time it takes all the sand to pass from the top half of the timer to the bottom half
Move	The setting of the Goal or the transfer of a cube from the Resources to Required, Permitted, or Forbidden
Mover	The player who made the most recent move
Negative number	A number smaller than 0
"Never"	The word formerly used when making an Impossible challenge
Never challenge	Same as an Impossible challenge

"No Goal"	The words the Goal-setter must use to make a no Goal declaration
No Goal challenge	The claim by an opponent that the no Goal declaration of the Goal-setter is incorrect; section **III-D** of the On-Sets Tournament Rules explains how to work out this challenge
No Goal declaration	The claim by the Goal-setter that no Goal can be set which has a possible Solution from the remaining Resources
"Now"	The word a player says when making a Now challenge
Now challenge	A challenge that a Solution can be written using all the cubes in Required, none, some, or all the cubes in Permitted, and, if needed, one cube in Resources
Null set	Same as empty set
Numeral cube	Same as digit cube
Operation cube	A cube with an operation symbol on every face
Operation symbol	One of the symbols $\underline{U}$, $\underline{\Omega}$, –, or '; this definition applies whether these symbols have their usual mathematical meanings or special meanings defined by variations
Order of operations	The sequence in which operations are to be performed in the Goal or a Solution
Penalty	The punishment a player receives for an illegal action; examples: loss of one point, returning an illegally played cube to Resources
Permitted	The section of the playing mat labeled PERMITTED; cubes in this section may be used in any Solution
Playing mat	The rectangular piece of cardboard (or, in older games, two rectangular cardboards) that contains the REQUIRED, PERMITTED, FORBIDDEN, and GOAL sections; even if the physical mat contains a RESOURCES section, this is not considered part of the playing mat
Presenting a Solution	Submitting your Solution to be checked by the opponent(s) by handing it to an opponent, placing it in the middle of the playing area, or allowing an opponent take it from you.
Regular move	A move that is not a bonus move; that is, play of a cube from Resources to the mat without calling "Bonus" or the second move on a turn after a bonus move
Required	The section of the playing mat labeled REQUIRED; cubes in this section must be used in any Solution
Resource cube	A cube that has been rolled for the shake but has not yet been played to the Goal, Required, Permitted, or Forbidden sections

Resources	The symbols that appear on the top faces of the cubes after they have been rolled for the shake
Restricted Universe	The Advanced On-Sets Universe after the Restriction part of a Solution has been applied to the original Universe
Restriction	In Advanced On-Sets, a statement containing one or more = or C̲ symbols
Restriction cube	A cube containing the symbols =, C̲, V̲, and /\̲ on its faces
Restriction part	That part of an Advanced On-Sets Solution consisting of one or more Restrictions
Restriction statement	Same as Restriction
Round	Same as Match
Scoresheet	A piece of paper on which players' scores for the shakes of a match are recorded and totaled and which the players sign or initial and which the winner of the match (or one of the winners if there is a tie) turns in to a designated person or place when the match ends
Senior Division	The division of play for students in grade 12 and below
Set-Name	An expression which names a set of zero or more cards from the Universe
Set-Name part	That part of a Solution which consists of one Set-Name
Set Subtraction	The operation represented by the symbol –; *Set1 – Set2* produces the cards that are in *Set1* but not in *Set2*
Shake	A play of the game starting with the roll of the cubes and a deal of the cards and ending with at least one player writing a Solution
Solution	A Set-Name part and, if necessary in Advanced On-Sets, a Restriction part
Solution-writer	A player who must present a Solution after a challenge or after the last Resource cube is moved to the playing mat
Subtraction	Same as Set subtraction
Symmetric difference	The operation defined by the Symmetric difference variation for the symbol –; *Set1 – Set2* produces the cards that are in either *Set1* or *Set2* but not in both
Table	A group of two or three players competing against each other in a match
Ten-second warning	The countdown from 10 to 1 that an opponent must make out loud when the sand runs out on a player's time limit

Third Party	In a three-player match in which a challenge has been made, the player who is neither the Challenger nor the Mover
Timed player	A player who is being timed to complete a task (roll the cubes, set a Goal, make a move, write or check a Solution, etc.)
Time limit	The amount of time allotted by the rules for a player to complete an action
Timer	An hour-glass-shaped piece of plastic (or glass) containing sand that is used to determine one- and two-minute time limits during a match; the sand in a timer usually takes about 60 seconds to pass through
Turn	The time when a player is obligated to move (which includes setting the Goal), challenge, or complete a required action (e.g., selecting a variation) within some specified time limit
Union	The operation represented by the symbol U; the union of two sets is the set of cards that are in *either* set or both
Universal set	The set of all cards dealt for a shake
Universe	Same as Universal set
Universe chart	A chart consisting of squares in which the cards in the Universe can be recorded
Valid challenge	A challenge that is not illegal
Variation	Same as Adventurous variation
Variation selection	A player's choice of one or two variations from the appropriate list; the player indicates the selection by circling the name of the variation(s) on the variation selection chart and, if necessary, filling in the blank to indicate which cube is wild or required, etc.
Variation selection chart	A chart printed either on the back of the scoresheet or on a separate page on which each player must record his/her variation selection for each shake on this chart
Warning	The announcement that the time limit for the match (round) has been reached and that no new shakes are to be started
Wild cube	A cube containing a symbol on its top face which may represent another symbol because of a variation chosen for the shake

Last update: 7/1/19