

EQUATIONS WORKSHEET

7X

NAME _____

FACTORIAL – II (EM only)

PRINCIPLE

The **factorial** variation works in combination with other variations.

Factorial in Combination with Other Variations

- (a) Sideways $\infty!$ is not defined. However, $(\infty \times 8)! = 4! = 4 \times 3 \times 2 = 24$
- (b) Upside down $\bar{z}!$ is undefined. However, $(\bar{z} + 6)! = 4! = 24$
- (c) Multiple ops. Only *two* ! may be used in a Solution since ! is not on the cubes and therefore cannot appear in Required or Permitted to be used multiple times.
- (d) Percent $5! \text{ } ^{\wedge} 20 = 120\% \text{ of } 20 = 1.20 \times 20 = 24$
- (e) Decimal pt. * as decimal point takes precedence over all other operations. So $4*3!$ may *not* be interpreted as $4.(3!)$ or 4.6 . ($4*3!$ has no defined interpretation.)
- (f) # factors $x(6!)$ is not allowed since $6!$ is bigger than 200. However $x(5!) = x120 = x(2^3 \times 3^1 \times 5^1) = 4 \times 2 \times 2 = 16$.
- (g) Small. prime $x(6!)$ is not allowed since $6!$ is bigger than 200. However $x(5!) = x120 = 127$.
- (h) E: 2-dig.num. In the Goal or a Solution, $12! \div (10!) = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 12 \times 11 = 132$
- (i) EI: 3-op. Sol. Each ! sign in a Solution counts as an operation *sign*. So the Solution $6! \div 2!$ contains three operations.
- (j) EI: LCM $6! \sqrt{(5!)} = 720 \sqrt{120} = 720$. In general, if $m > n$ (m and n whole numbers), then $m! \sqrt{n!} = m!$
- (k) EI: GCF $6! * (5!) = 720 * 120 = 120$. In general, if $m > n$ (m and n whole numbers), then $m! \sqrt{n!} = n!$
- (l) Mid: Mult.of k If $k = 6$, then $3!$, $4!$, $5!$, and so on, all equal 0 since each of these factorials contains factors of 2 and 3. If $k = 7$, $7!$, $8!$, $9!$, and so on equal 0 since each contains 7 as a factor. In general, $k!$, $(k+1)!$, $(k+2)!$, and so on, all equal 0. However, for non-prime values of k , factorials smaller than $k!$ may also be 0.
- (m) Mid: Red ex. A Goal of 23 (red 3) may be interpreted as $2^3!$ or 2^6 . In a Solution write $2^{(3!)}$ to prevent an opponent from interpreting the expression as $(2^3)!$

EXERCISES

Give the value of each interpretation of each expression. Assume factorial is in effect along with the variation listed.

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|------------------|-----------------------|-----------------|------------------|
| 1. sideways | $(6 \times \infty)!$ | 2. upside-down | $(4 - \bar{z})!$ |
| 3. decimal pt. | $4! \times 1*5$ | 4. # factors | $x(4!)$ |
| 5. # factors | $x3!$ | 6. small. prime | $x(4!)$ |
| 7. small. prime | $x3!$ | 8. average | $3! + (5!)$ |
| 9. E: 2-dig.num. | $11! \div (8!)$ | 10. E: LCM | $5! \sqrt{(4!)}$ |
| 11. E: GCF | $4! * (6!)$ | 12. M: base 8 | $12! \div (10!)$ |
| 13. M: base 9 | $11! \div [(5 + 4)!]$ | 14. M: red exp. | $3^3!$ |

Middle: Give the smallest non-negative value of each factorial for the given multiple of k .

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|------------------|-------|-------------------|-------|------------------|-------|
| 15. $k = 8, 9!$ | _____ | 16. $k = 9, 6!$ | _____ | 17. $k = 7, 7!$ | _____ |
| 18. $k = 10, 5!$ | _____ | 19. $k = 11, 43!$ | _____ | 20. $k = 12, 4!$ | _____ |