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## REMAINDER (EM ONLY) - I

## PRINCIPLE

The Remainder variation says: " $A \cdot B$ ( $\cdot$ is a sideways $\div$ ) equals the remainder when $A$ is divided by $B$. $A$ and $B$ are positive integers, and $A$ is less than or equal to 1000."

The phrase " $A$ is less than or equal to 1000 " does not mean you may use a three-digit number for $A$ (unless Decimal Point also in play). It means the value of $A$ can be no bigger than 1000.

## EXAMPLES

1. $15 \cdots 2=1$ since 15 divided by 2 gives a quotient of 7 with remainder 1 .
2. $(30 * 2) \cdot 1 \cdot 5=0$, the remainder when 900 is divided by 5 .
3. $45 \cdots 70=45$, since 70 goes into 45 zero times with 45 remainder.
4. $(32 * 2) \cdot 6$ is not allowed since $32^{*} 2$ is 1024 , which is over the 1000 limit.
5. $6 \cdot(32 * 2)=6$. There is no limit on the value of $B$, the number after the $\cdot \| \cdot$ sign.
6. $(6 \cdot 32)^{*} 2=6 * 2=36$.
7. The Goal $5-9 \cdot I \cdot 13$ has only one value.
a. $5-(9 \cdot 13)=5-9=-4$.
b. $(5-9) \cdot \| \cdot 13$ is not allowed since $5-9$ is negative.
8. The Goal $5^{*} 27 \cdot 1 \cdot 4$ has only one value.
a. 5 * $(27 \cdot 4)=5$ * $3=125$.
b. $\left(5^{*} 27\right) \cdot 4$ is not allowed since 5 * 27 is far beyond the 1000 limit.

## EXERCISES

With Remainder, write all values of each Goal.
Goal Values Goal

Values

1. 7 小 3
2. $4 \% 9$
$\qquad$ 2. $\quad 10 \div 4$
3. $\quad 17 \cdot 6$
4. $5 \cdots 18$
5. $\quad 38 \cdot 2$
6. $97 \cdot 1 \cdot 10$
7. $5 * 3 \cdot \mid \cdot 4$
8. $9 \cdot \mid \cdot 2+13$
9. $29 \cdot \mid \cdot 5-8$
10. $83 \cdot|\cdot 7 \cdot| \cdot 5$ $\qquad$
11. $7 * 3 \cdot 1 \cdot 5$ $\qquad$
12. $6 * 3 \cdot \mid \cdot 10$ $\qquad$

## EQUATIONS WORKSHEET

NAME $\qquad$

## REMAINDER (EM ONLY) - II

## PRINCIPLE

The Remainder variation may be used with other variations.

## EXAMPLES

1. Upside-down: Neither the number before the $\cdot \|$ sign nor the number after the $\cdot \| \cdot$ sign may be negative.
2. 0 wild: $15 \% 0=0$ (when 0 is 1,3 , or 5 ), 1 (when 0 is 2 or 7 ), 3 (when 0 is 4 or 6 ), 6 (when 0 is 9 ), or 7 (when 0 is 8 ).
3. Factorial: $6!\cdots 5=0$ since 5 is one of the factors of $6!6 \% 5!=6$ since $5!=120$. $6!\cdot 1 \cdot 5!=0$ since 5 ! is included in 6 !
4. Multiple Operations: Any $\div$ cube may be used multiple times in the Solution. Each $\div$ sign in the Solution may be either division or remainder. Middle: If 0 wild is also chosen, any 0 may be used multiple times as either $\div$ or $\cdots$.
5. Three-operation Solution: $\cdot$ counts as an operation.

## EXERCISES

With Remainder and the variation listed chosen, write all values of each Goal.

|  | Variation | Goal | $\underline{\text { Values }}$ |
| :---: | :---: | :---: | :---: |
| 1. | Sideways | $7 \div 5$ |  |
| 2. | 0 wild | $24 \cdot 0$ |  |
| 3. | Factorial | $5 \cdots 3$ |  |
| 4. | Percent | 9-1-25-48 |  |
| 5. | Dec. Point | $135 * \cdot 1 \cdot 7$ |  |
| 6. | Dec. Point | 999*-1/5 |  |

\#7-8 are for Elementary Division only.
7. Smallest Prime $x 32 \cdots 6$
8. Smallest Prime $\quad x 99 \cdot \mid \cdot 7$
\#9-10 are for Middle Division only.
9. Red Exponent $72 \cdot 15($ red 2$)$ $\qquad$
10. Powers of Base $\quad 1 \% 6$

## MORE CHALLENGING EXERCISES - MIDDLE DIVISION

In \#11-13, $x$ and $y$ positive whole numbers.
11. If $x \geq y$, then $x$ ! $\cdot y$ ! $=$ $\qquad$
12. If $x \geq y$, then $x$ ! $\cdot y=$ $\qquad$
13. If $x<y$, then $x \cdot \mid \cdot y$ ! $=$ $\qquad$

## EQUATIONS WORKSHEET

NAME $\qquad$

## REMAINDER (EM ONLY) - III

## PRINCIPLE

The Remainder variation may be used to pad Solutions.

## EXAMPLES

1. Since $3 \cdots 4=3,3 \cdot 5=3,3 \cdot 6=3$, etc., you can pad Solutions like this.

2. The remainder when you divide any whole number by 1 is 0 . So you can pad Solutions like this.
Equation: $(7 \times 4)+0=28$

Padded Equation: $(7 \times 4)+(\square$
Any positive whole number
3. The remainder when you divide any positive whole number by itself is 0 . Also, the remainder is 0 when the number before the $\cdot$ is a multiple of the number behind the $\%$. Use this fact to pad Solutions like this.
Equation: $(7 \times 4)-0=28$
Padded Equation: $(7 \times 4)-(\underset{\uparrow}{\ldots})=28$
Same number in both places or combinations like $4 \cdot 1 \cdot 2,9 \cdot I \cdot 3$, etc.
4. $9 \cdots 8=1,8 \cdots 7=1,7 \cdots 6=1$, etc. Use this pattern to pad Solutions like this.

Equation: $(7 \times 4) \times 1=28$

Padded Equation: $(7 \times 4) \times \underset{\uparrow}{(8 \cdot 7)}=28$
One less than the number before the $\cdot \boldsymbol{F}$

## EXERCISES

With Remainder chosen, use all the Resources listed to make a Solution for each Goal.

Resources
Equation

1. $494569 \div+x$
2. 35
$13459+-x \div$
$3 . \quad 69$
$23589++$ *
3. 54
$36799+x \times \div$

## MORE CHALLENGING EXERCISES - MIDDLE DIVISION

In each case, $x$ is a positive whole number.
5. $x \cdot 1 \cdot x=$ $\qquad$ 6. $(x+1) \cdot 1 \cdot x=$ $\qquad$
7. $x \cdot 1=$ $\qquad$ 8. $\left(x^{\wedge} n\right) \cdot x=$ $\qquad$ ( $n=$ positive whole \#)

## SOLUTION KEY

## worksheet R1

1. 1
2. 2
3. 4
4. 5
5. 3
6. 5
7. 1
8. 0
9. 1
10.7
11.0, 8
worksheet R2
10. 2
11. $0,3,4,6$
12. $0,5,40$
13. $0.72,1$
14. 2
15. 4
16. 1,3
12.1, 125
13.6, 8
14.9, 14
15.15. 4
17. -4
17.0, 71
18.1, 2
19.0, 512
20.3, 343
21.0, 12
22.6, 216
18. 2,3
19. 2,4
10.1, 4
11.0
12.0
20. $X$

## WORKSHEET R3

1. $(5 \times 9)+(4 \cdot 1 \cdot 6)$
2. $[(4+3) \times 5]-(9 \cdot 1)$
3. $[(8$ * 2$)+5]+(9 \cdot I \cdot 3)$
4. $(6 \times 9) \times[(3+7) \cdot 9]$
5. 0
6. 1
7. 0
8. 0
